EXPERIMENTAL METHODS

FOR IN-DUCT AEROACOUSTIC MEASUREMENTS

KTH Aeronautical
and Vehicle Engineering

Hans Bodén

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INTRODUCTION

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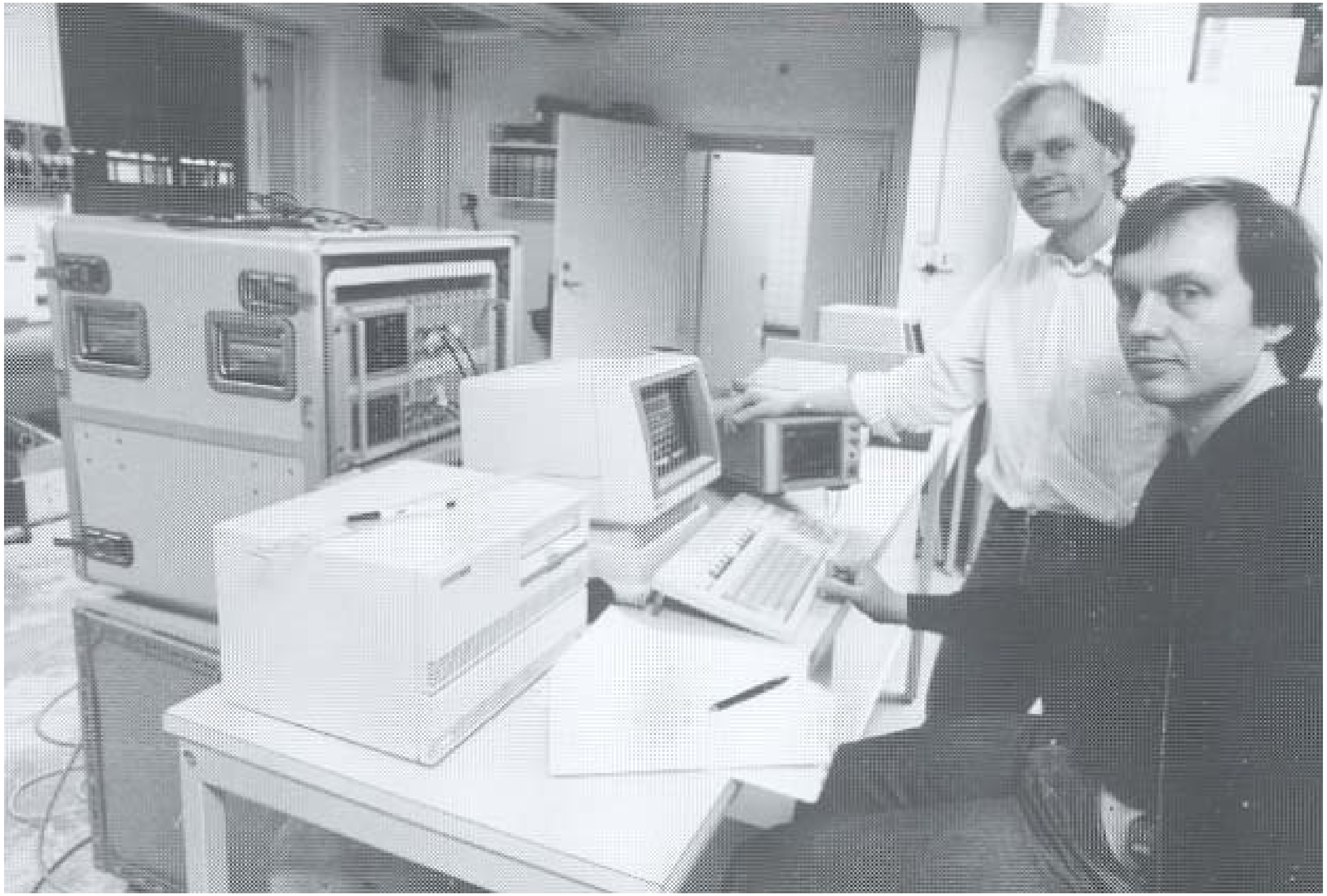

1986?

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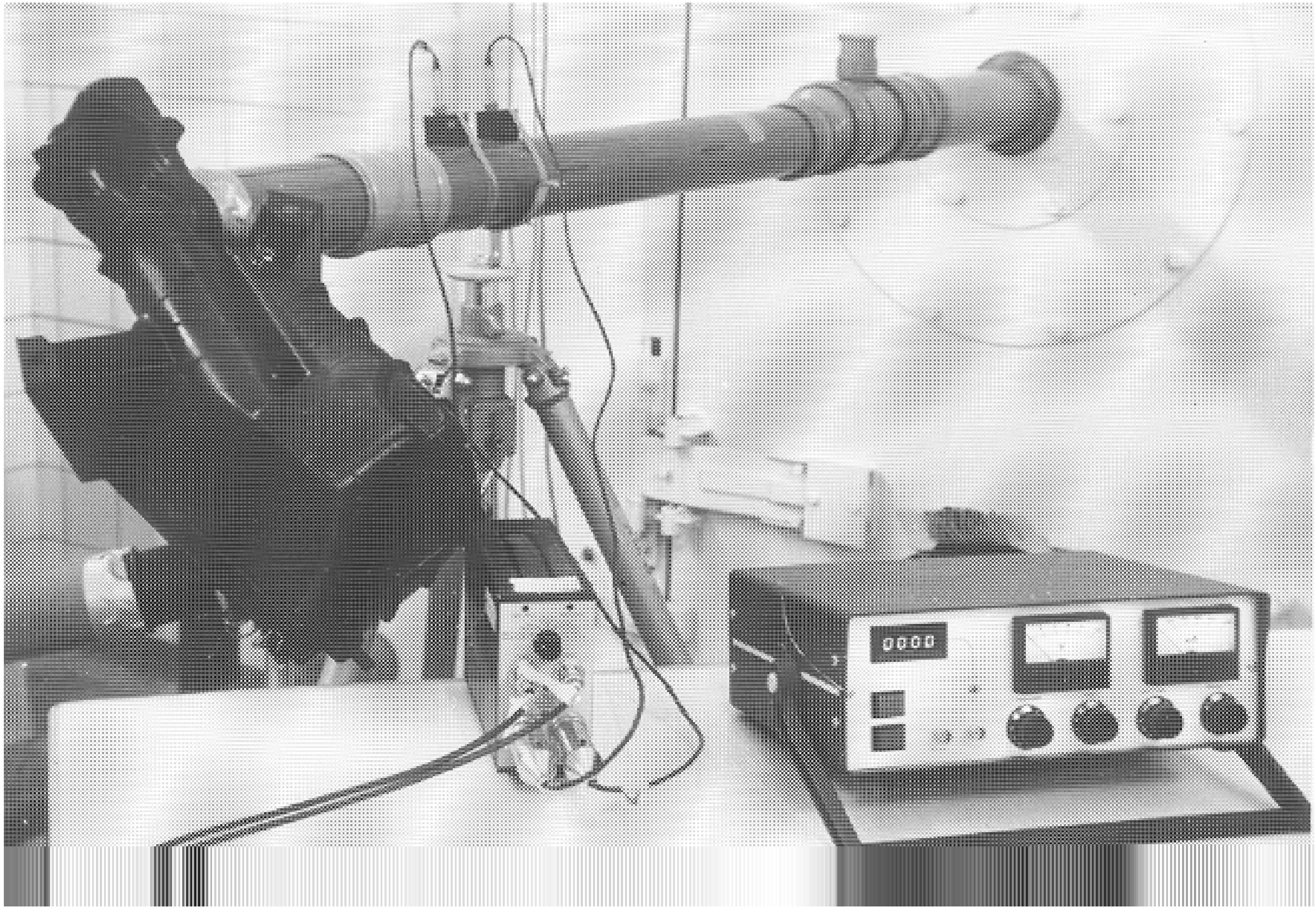

TANGO Workshop in aero-acoustics in confined flows of low Mach number, KTH 2014-05-22

Influence of errors on the two-microphone method for measuring acoustic properties in ducts

Hans Bodén and Mats Åbom
Department of Technical Acoustics, Royal Institute of Technology, 100 44 Stockholm, Sweden

(Received 6 December 1984; accepted for publication 9 September 1985)

Using the two-microphone method, acoustic properties in ducts, as, for example, reflection coefficient and acoustic impedance, can be calculated from a transfer function measurement between two microphones. In this paper, a systematic investigation of the various measurement errors that can occur and their effect on the calculated quantities is made. The input data for the calculations are the measured transfer function, the microphone separation, and the distance between one microphone and the sample. First, errors in the estimate of the transfer function are treated. Conclusions concerning the most favorable measurement configuration to avoid these errors are drawn. Next, the length measurement errors are treated. Measurements were made to study the question of microphone interference. The influence of errors on the calculated quantities has been investigated by numerical simulation. From this, conclusions are drawn on the useful frequency range for a given microphone separation and on the magnitude of errors to expect for different cases.

PACS numbers: 43.85.Vb, 43.85.Bh, 43.88.Kb, 43.55.Ev

541 J. Acoust. Soc. Am. 79 (2), February 1986 0001-4966/86/020541-09$00.80 541

Error analysis of two-microphone measurements in ducts with flow

Mats Åbom and Hans Bodén
Department of Technical Acoustics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

(Received 2 February 1987; accepted for publication 18 January 1988)

In an earlier work [H. Bodén and M. Åbom, J. Acoust. Soc. Am. **79**, 541–549 (1986)] the influence of errors on two-microphone measurements in ducts without flow has been studied. The aim of this article is mainly to extend the earlier work to include the effects of mean flow and also of attenuation during the sound propagation. First, a short review of the various existing two-microphone methods is made. The errors in the measured input data are then analyzed and special attention is paid to the effects of neglected attenuation, nonideal microphones, and flow noise. The influence of errors on the calculated quantities has been investigated and the conclusions from the earlier work have been extended to the case with flow. It is also shown that the neglect of attenuation between the microphones leads to a low-frequency limit for the applicability of the two-microphone method. Finally, a new technique for measuring the Mach number using a two-microphone method is suggested.

PACS numbers: 43.88.Kb, 47.60. + i, 43.85.Vb, 43.50.Nm

2429 J. Acoust. Soc. Am. **83** (6), June 1988 0001-4966/88/062429-10$00.80 2429

Passive one-port measurement techniques
The ”Two-Microphone Method”

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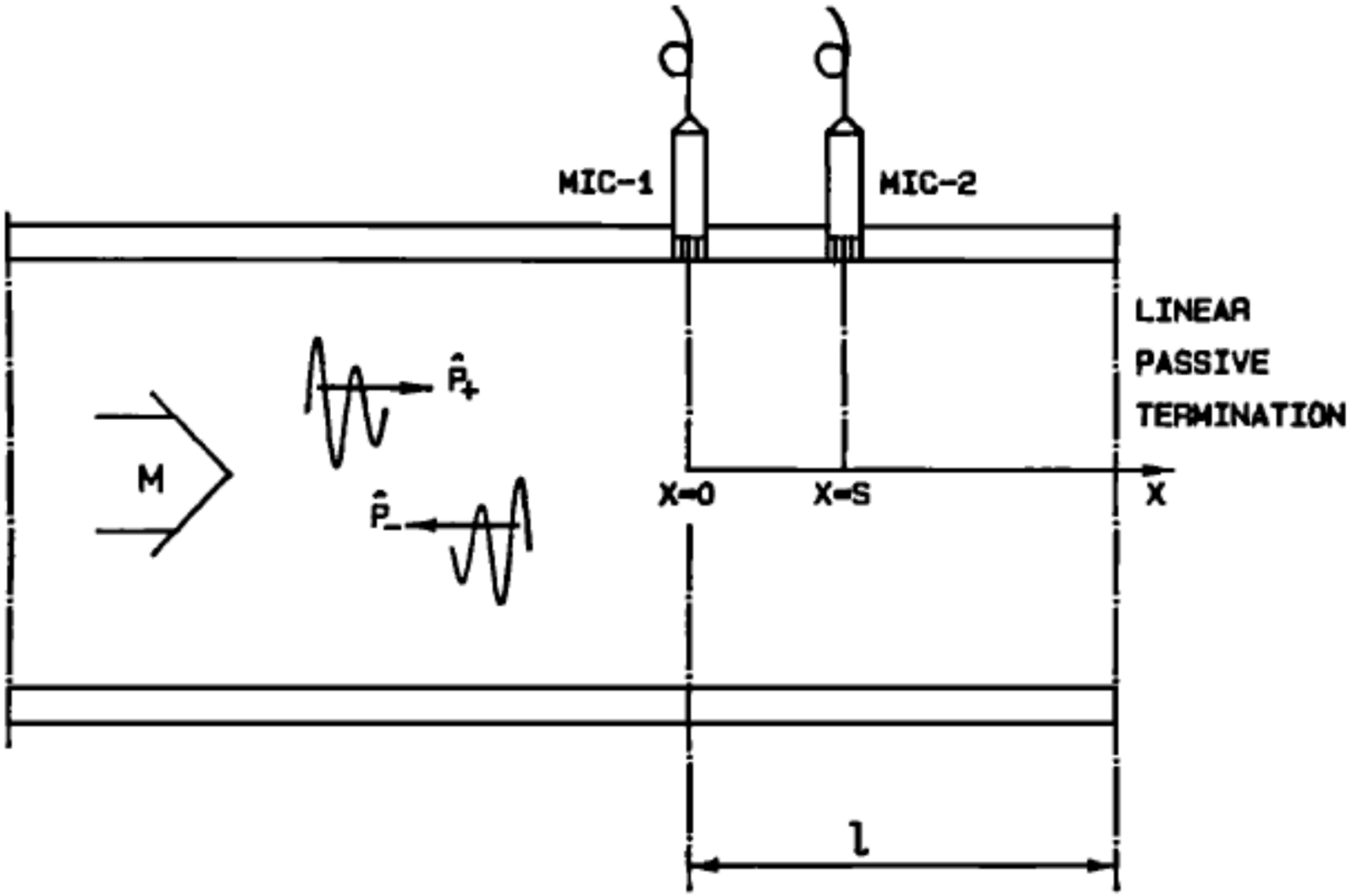

The Two-Microphone Method

$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f) \cdot \exp(-jk_+ s) + p_-(f) \cdot \exp(jk_- s)$$

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where $k_+ = \dfrac{k}{1+M}$ $\quad k_- = \dfrac{k}{1-M}$

A linear system of equations in p_+ and p_- from which the **Reflection Coefficient** at x=0 can be calculated

$$R_0(f) = \frac{p_-(f)}{p_+(f)} = \frac{\exp(-jk_+ s) - {p_2(f)}/{p_1(f)}}{{p_2(f)}/{p_1(f)} - \exp(jk_- s)}$$

The Two-Microphone Method

With $H_{12}(f) = \dfrac{p_2(f)}{p_1(f)}$ we get

$$R_0(f) = \frac{\exp(-jk_+s) - H_{12}(f)}{H_{12}(f) - \exp(jk_-s)}$$

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The reflection coefficient at *x*=*l* can be calculated from

$$R_l(f) = R_0(f)\frac{\exp(j2kl)}{1-M^2}$$

And the normalised impedance (= one port passive system properties) can be calculated from

$$Z(f) = \frac{p(f)}{\rho_0 c \cdot u(f)} = \frac{1+R(f)}{1-R(f)}$$

Errors in the Two-Microphone Method

$$R_0(f) = \frac{\exp(-jk_+s) - H_{12}(f)}{H_{12}(f) - \exp(jk_-s)}$$

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Errors in the input data: k_+s, k_-s and $H_{12}(f)$

Errors in k_+s and k_-s:

•Uncertainty in determination of *k* because of mainly turbulent losses

•Uncertainty in Mach-number measurement

•Uncertainty in length measurement: geometric and acoustic length

Errors in the Two-Microphone Method

Errors in $H_{12}(f)$

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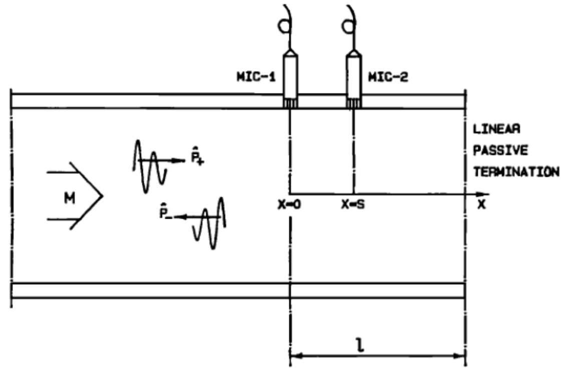
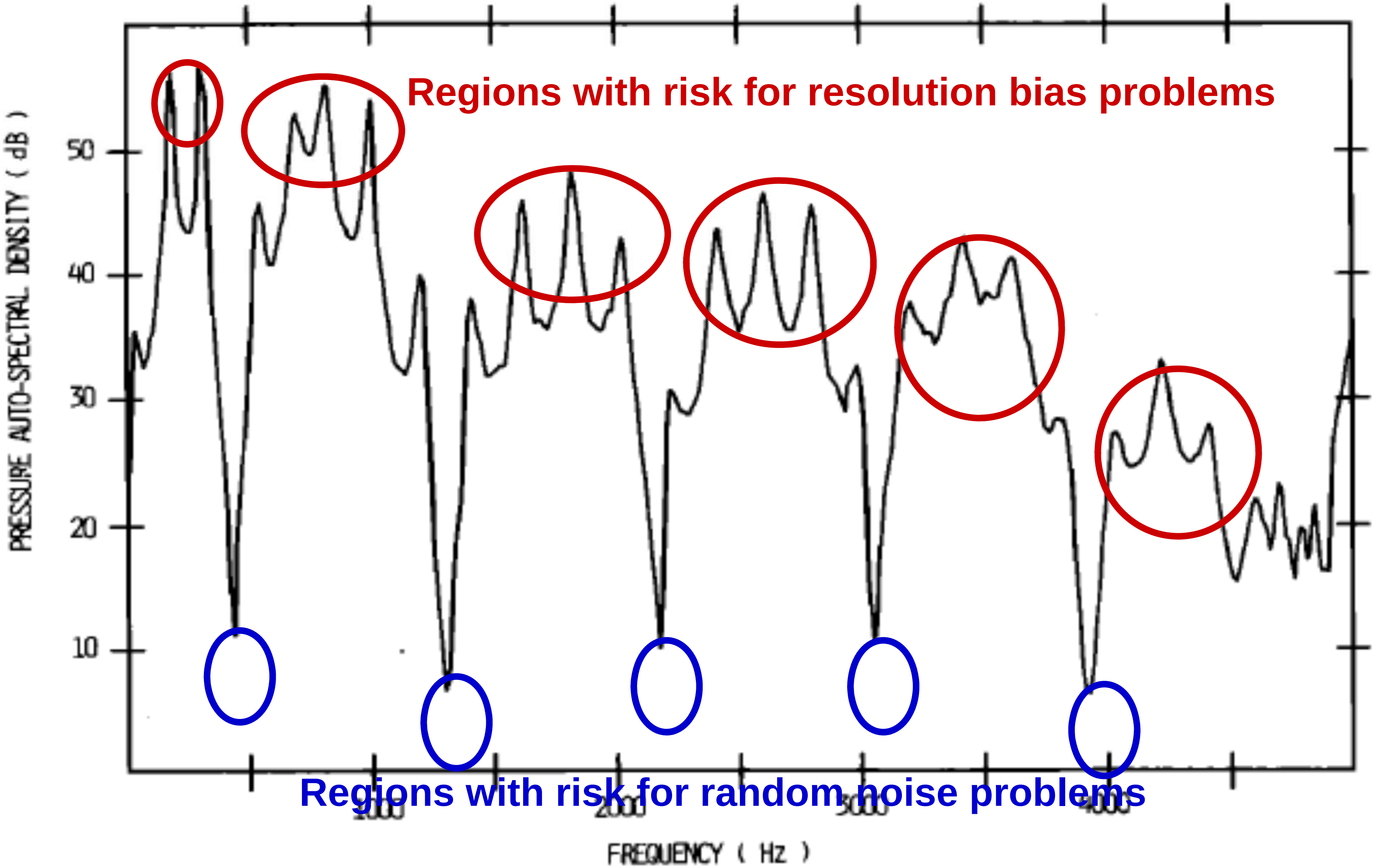
- Bias errorrs, as for instance resolution-bias errors. Problem for long duct systems with many resonances. Solution – Reflection free terminations.
- Random errors caused by random signals but mainly flow noise disturbances



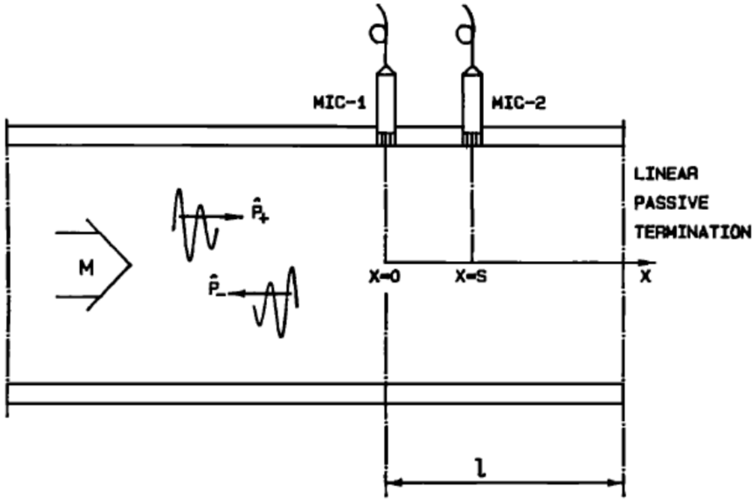
Pressure autospectral density measured in a duct driven by a loudspeaker and with a rigid termination.



Errors in the Two-Microphone Method

To avoid large sensitivity to the errors in the input data the two-microphone yechnique should be restricted to the frequency range:

$$0.1 \cdot \pi \cdot \left(1 - M^2\right) < ks < 0.8 \cdot \pi \cdot \left(1 - M^2\right)$$



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Calibration

Duct method for measurement of $K_{AB}(\omega)$.

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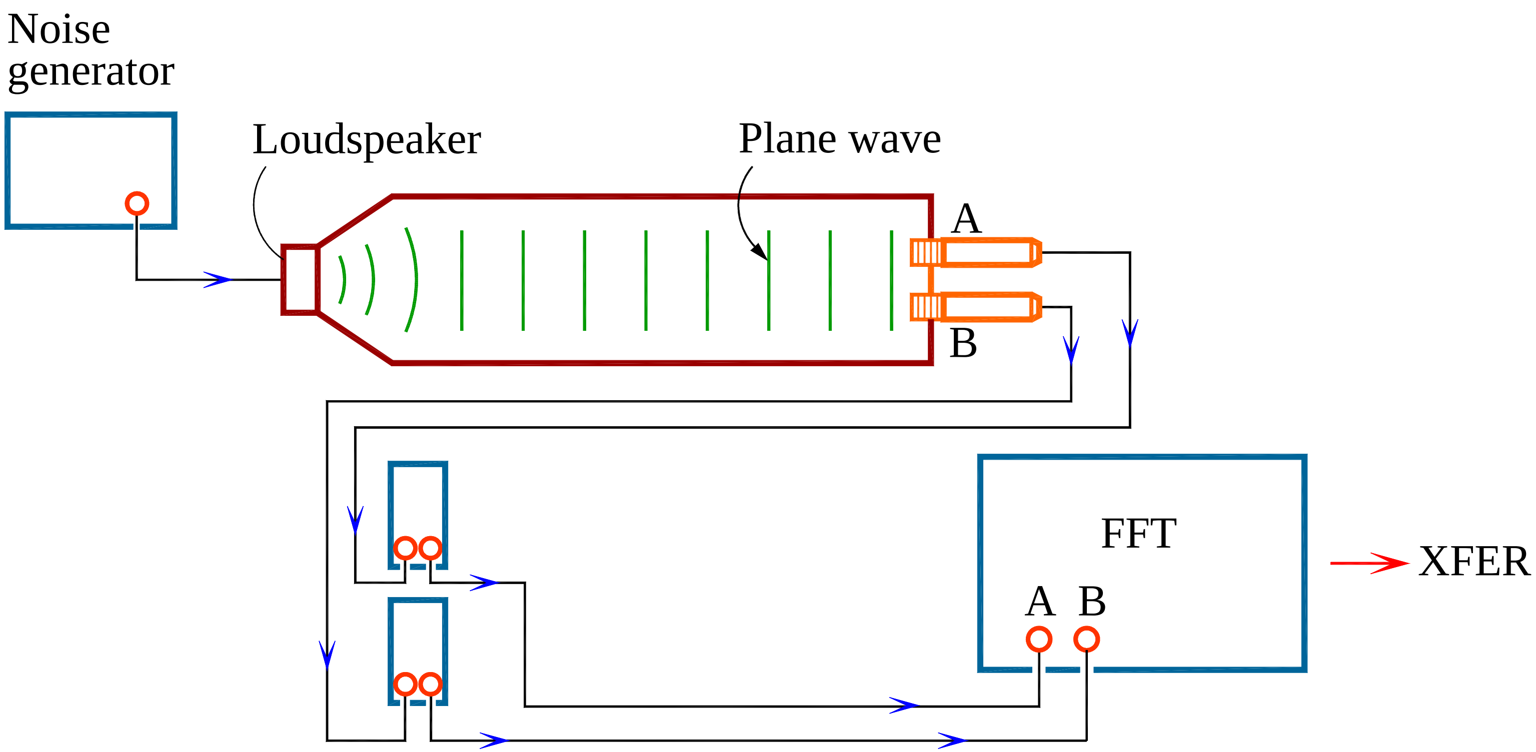

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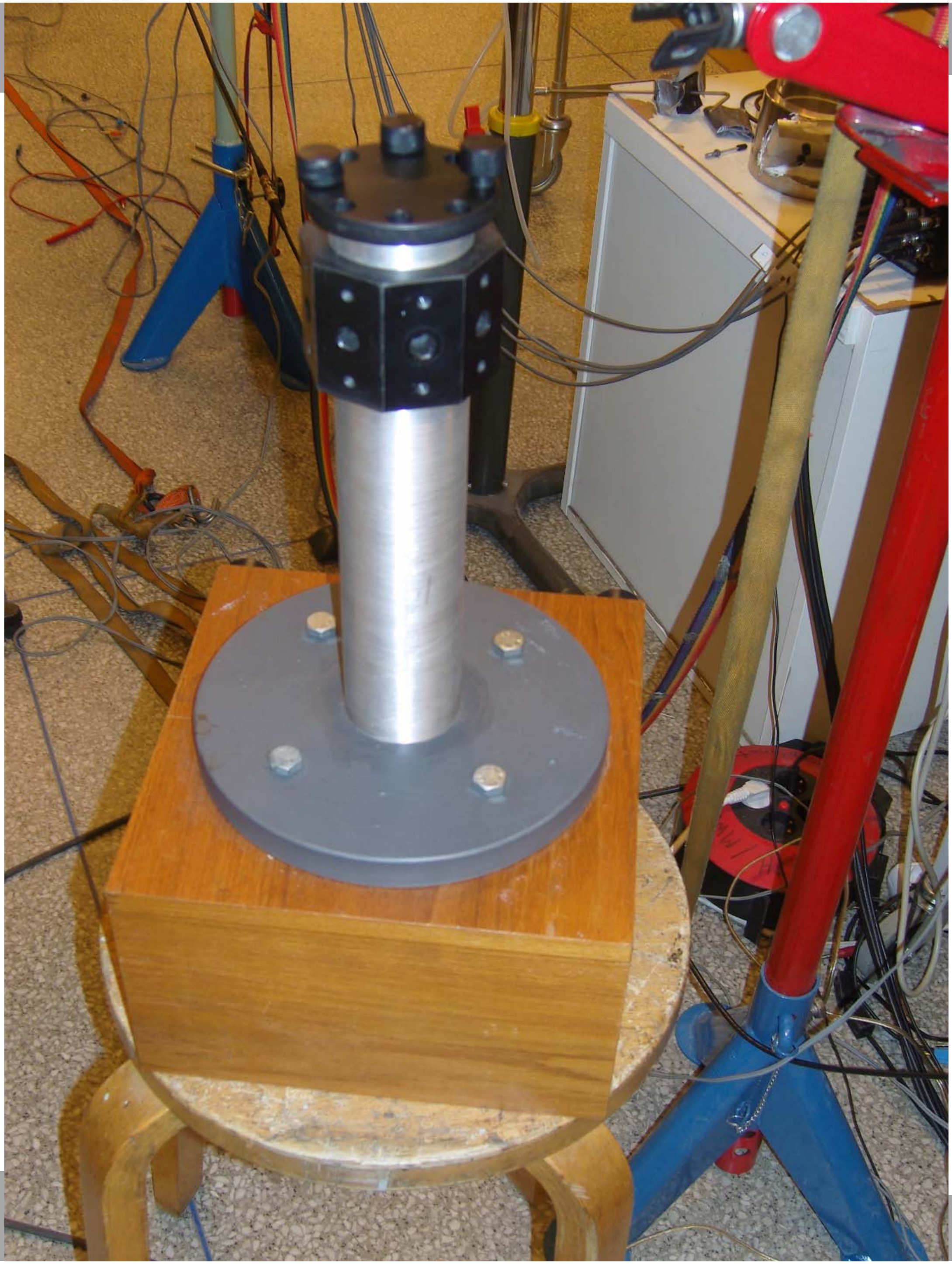

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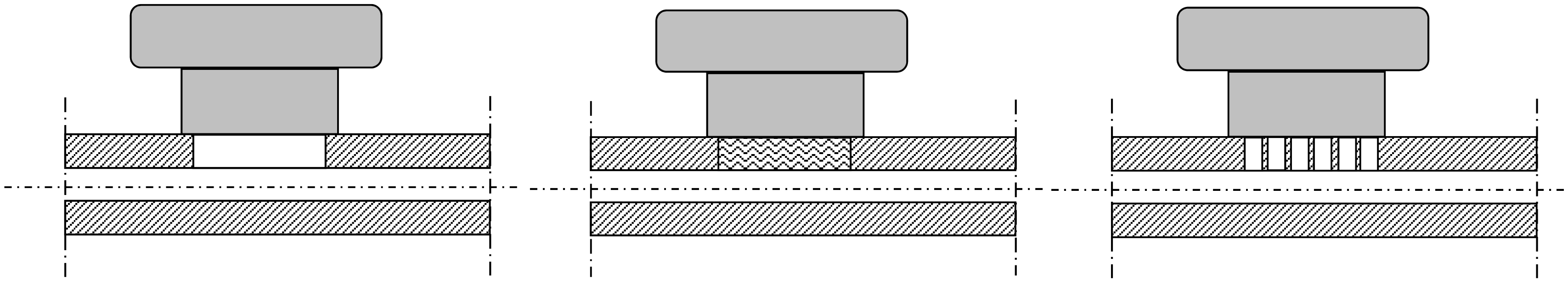
Effect of loudspeaker mounting configurations



a) loudspeaker connected using an open hole.

b) the hole filled with absorbing material

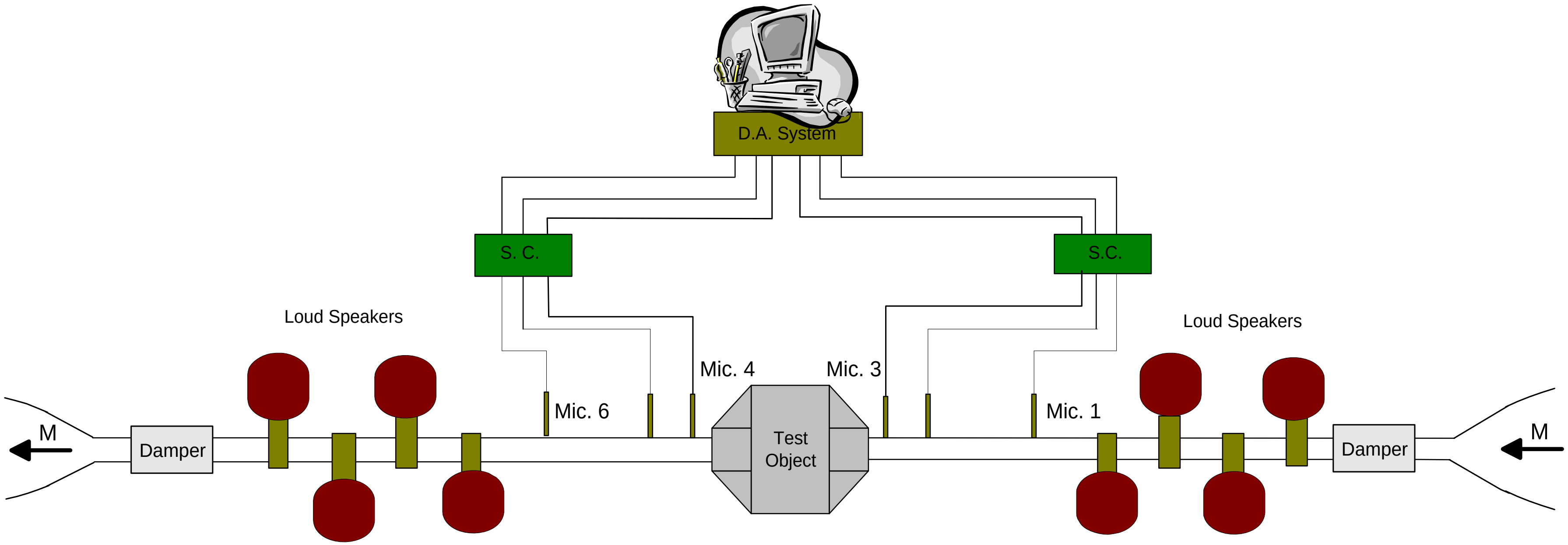
c) loudspeaker connected using a perforate pipe with 50% porosity.

Tests at $M = 0.15$ and 0.3 showed that configuration c) with a 50% perforate gave the best signal-to-noise ratio at the microphones

Signal to-noise ratio
P_S is the sound power of the acoustic signal and
P_N is the unwanted noise sound power.

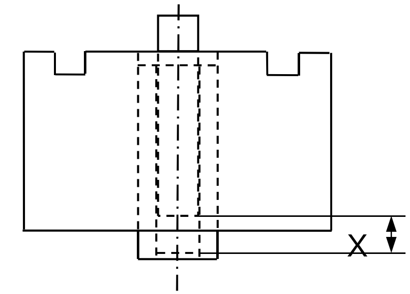
$$SNR = 10 \log_{10}\left(\frac{P_S}{P_N}\right)$$

Effect of loudspeaker mounting configurations

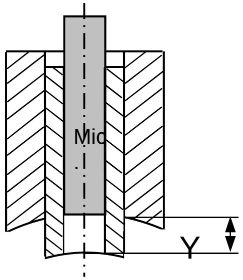


One should also avoid equidistant loudspeker separations which may cause cancellation at certain frequencies

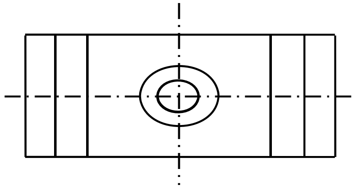
Effect of microphone holders



Elevation

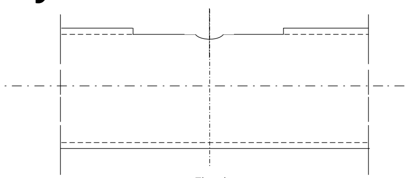


Side View Cross Section

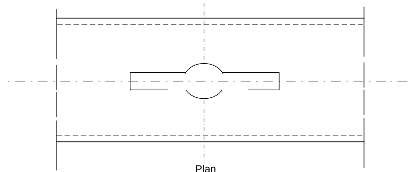


Plan

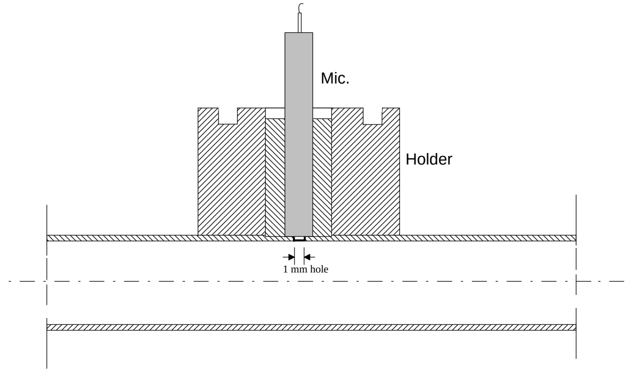
Reference holder with flush mounted microphone. Also used with the microphone moved 1.5 mm and 3 mm away from the duct wall



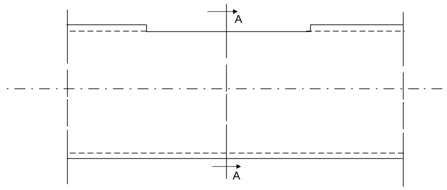
Elevation



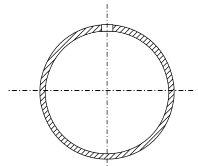
Plan



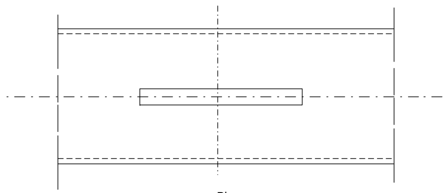
Microphone connected to duct via a 1 mm diameter hole.



Elevation



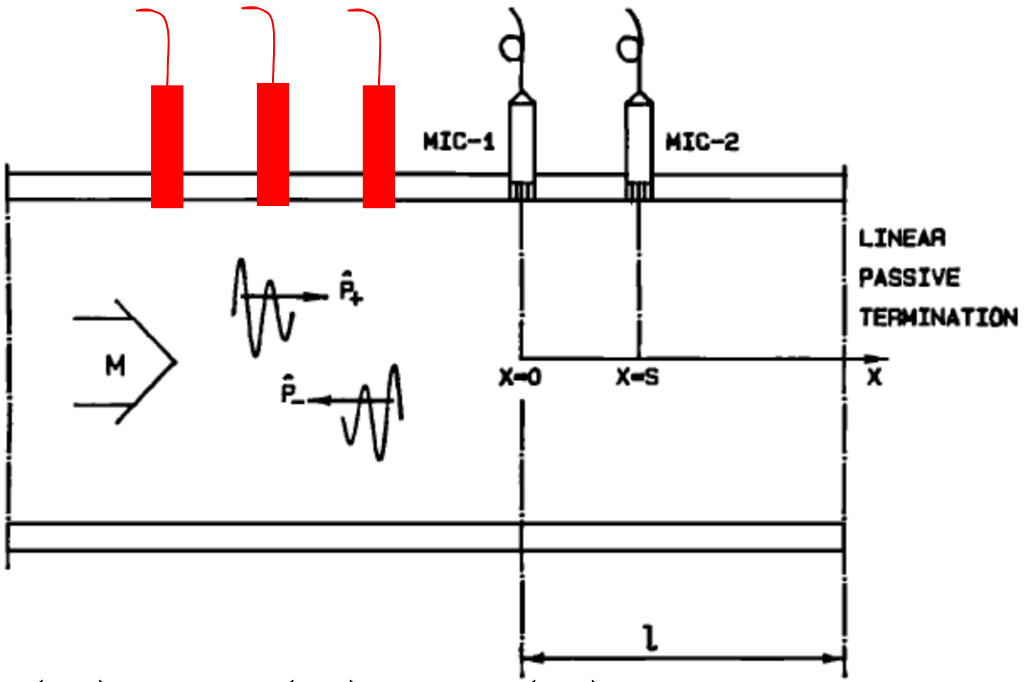
Side View
Cross Section at AA



Plan

Microphone connected to duct via a 30 mm long and 2 mm wide slit. Also a 6.4 mm diameter hole added to the slit.

Over-determination



Add more microphones

Can be used to reduce error

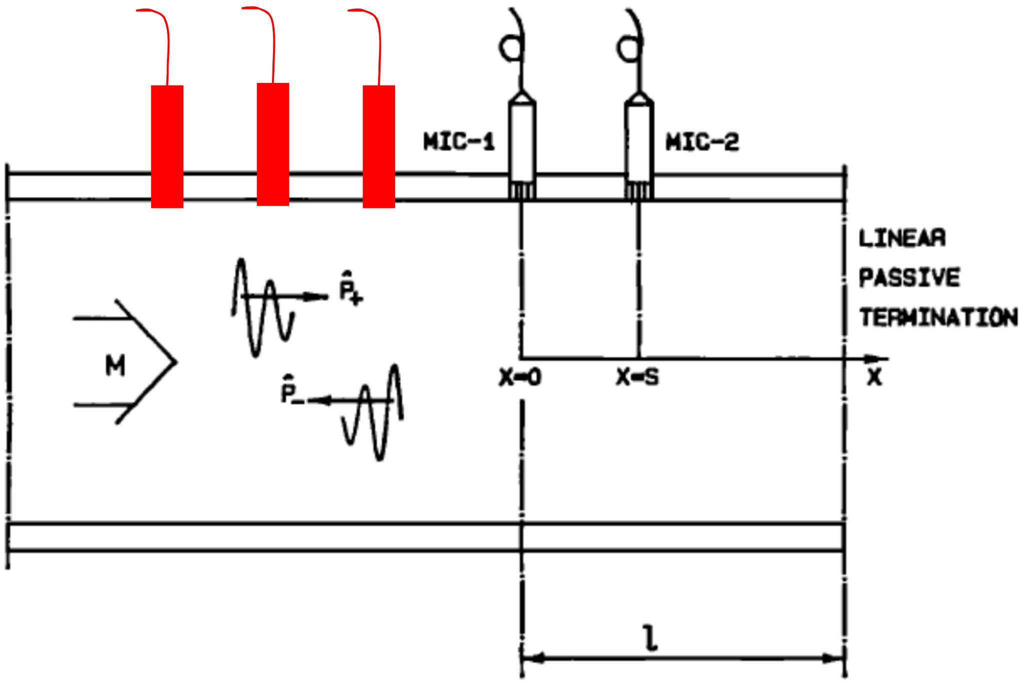
$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f)\cdot\exp(-jk_+s) + p_-(f)\cdot\exp(jk_-s)$$

$$\vdots$$

$$p_n(f) = p_+(f)\cdot\exp(-jk_+s_n) + p_-(f)\cdot\exp(jk_-s_n)$$

Over-determination

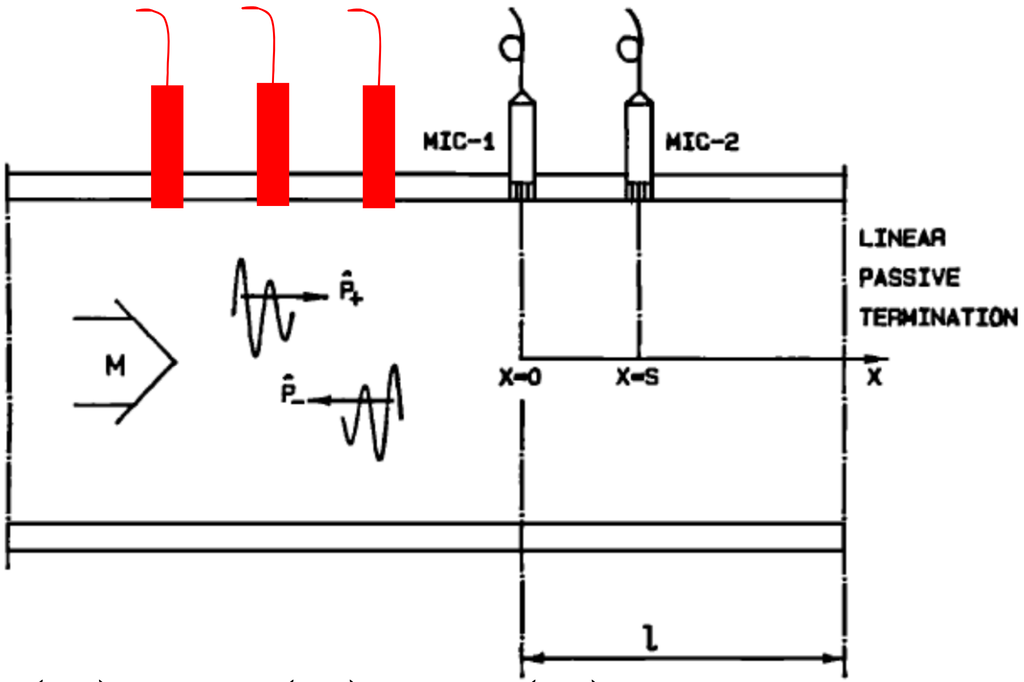


Add more microphones

Can be used to reduce error

Accurate experimental two-port analysis of flow generated sound
Andreas Holmberg, Mats Åbom and HansBodén,
Journal of Sound and Vibration 330 (2011) 6336–6354

Treat k_+s and k_-s as unknowns



Add more microphones

Solve nonlinear system of equations for *k_+s* and *k_-s*

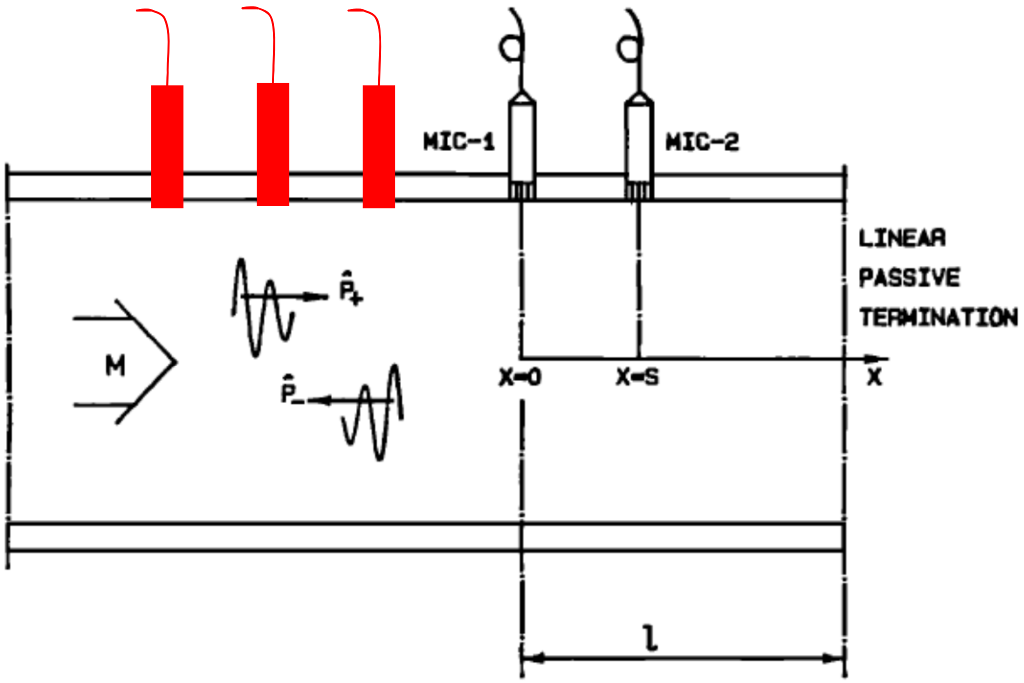
$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f)\cdot\exp(-jk_+s) + p_-(f)\cdot\exp(jk_-s)$$

$$\vdots$$

$$p_n(f) = p_+(f)\cdot\exp(-jk_+s_n) + p_-(f)\cdot\exp(jk_-s_n)$$

Treat k_+s and k_-s as unknowns

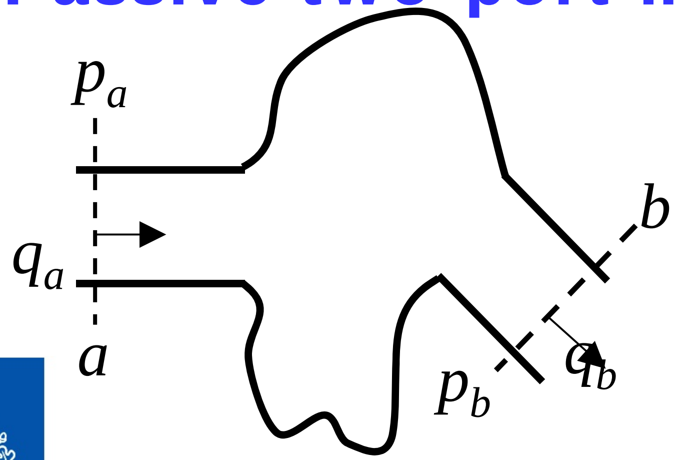


Add at least 4 microphones

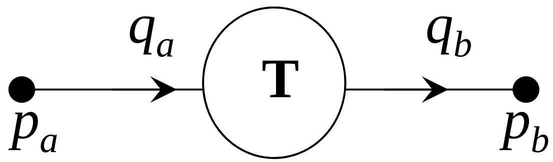
Solve nonlinear system of equations for k_+s and k_-s

S. Allam and M. Åbom, Investigation of damping and radiation using full plane wave decomposition in ducts. *Journal of Sound and Vibration* 292 (2006) 519-534. doi:10.1016/j.jsv.2005.08.016

Passive two-port measurement techniques



Physical system



Equivalent circuit

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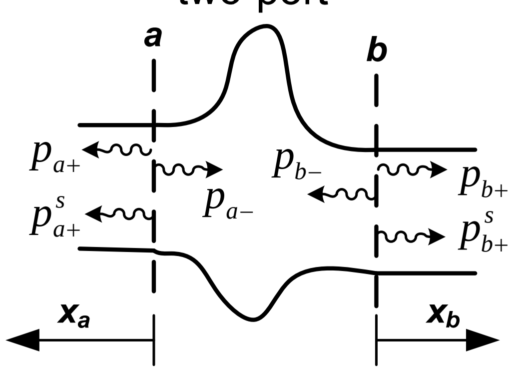
In the frequency domain a (linear) matrix relationship relates the states at *a* and *b.* A common choice of state variables is *p* and *q*.

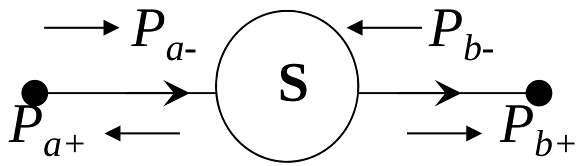
$$\begin{pmatrix} \hat{p}_a \\ \hat{q}_a \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \hat{p}_b \\ \hat{q}_b \end{pmatrix}$$

Mathematical model

”Four pole”



Physical system



Equivalent circuit

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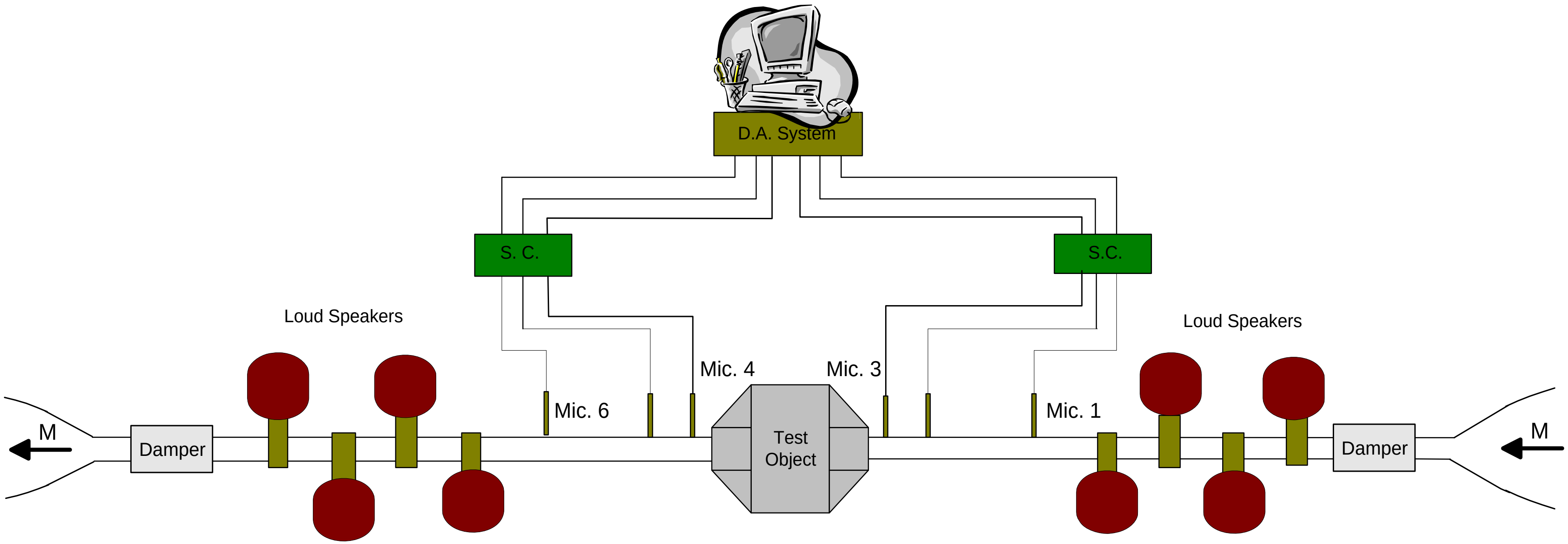
In the frequency domain a (linear) matrix relationship relates the states at *a* and *b*. Another common choice of state variables is p_+ and p_-.

$$\begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} p_{a+} \\ p_{b+} \end{pmatrix}$$

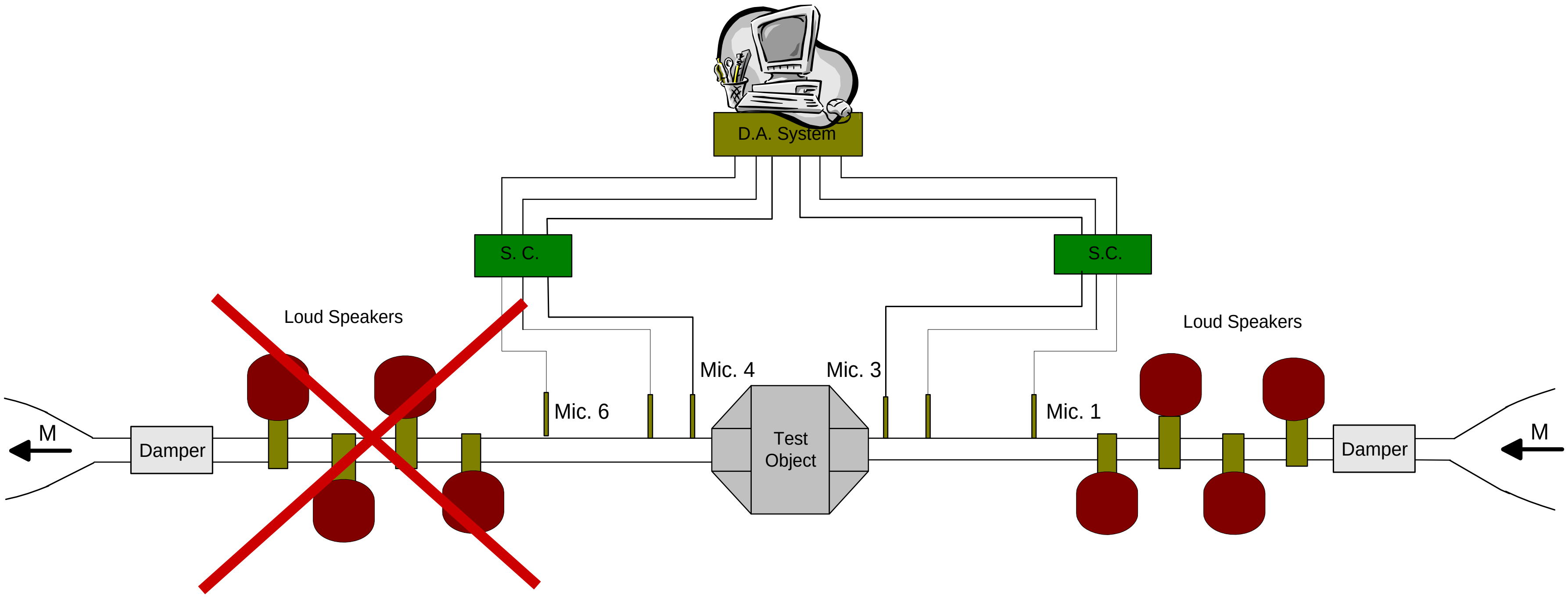
Mathematical model

”Scattering matrix”

Experimental determination of Two-port data using the Two-source technique



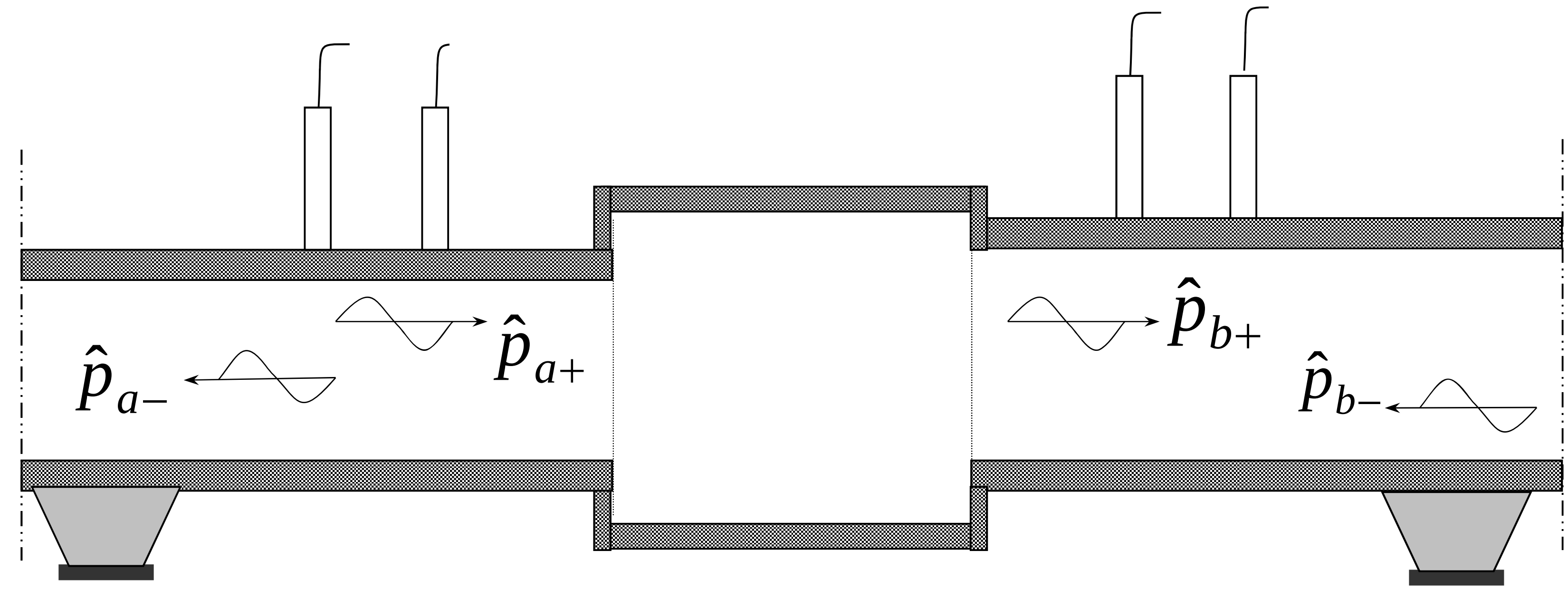
Experimental determination of Two-port data using the Two-load technique



Change acoustic load instead

Theoretical background

Two-port measurements using the two source technique



$$\begin{pmatrix} p_a \\ q_a \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b \\ q_b \end{pmatrix}$$

$$p_a = p_+ \exp\left(-i\, k_+^a L_a\right) + p_- \exp\left(i\, k_-^a L_a\right)$$

$$q_a = \frac{A_a}{\rho c} \left\{\left(p_+ \exp\left(-i\, k_+^a L_a\right) - p_- \exp\left(i\, k_-^a L_a\right) \right)\right\}$$

Theoretical background

$$\begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix}$$

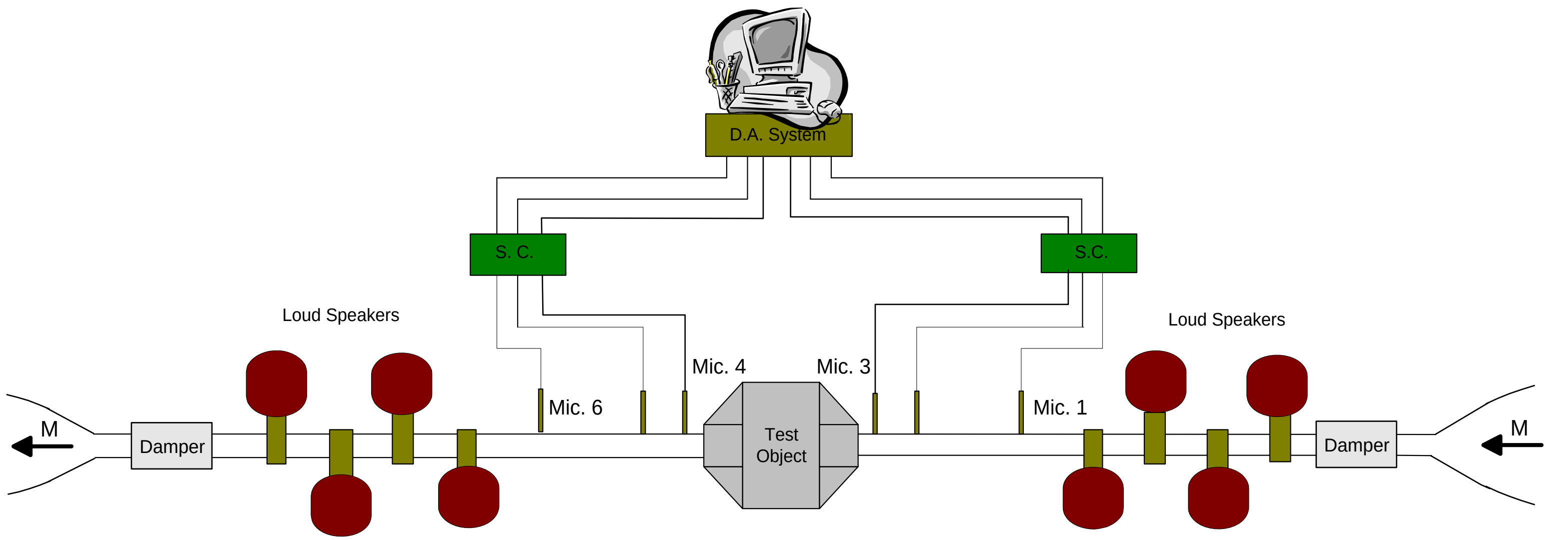
and the two-port matrix is determined from:

$$\begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} = \begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix}^{-1}$$

if

$$\det \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix} \neq 0$$

Typical test setup at MWL:



6-8 loudspeakers, two-source technique used, input signal available as reference

6-12 B&K ¼-inch microphones

Flow speed measured using Pitot tube and hotwire anemometer. Measurements made before and after the acoustic measurements.

Flow Noise Supression

Obtain initial good signal-to-noise ratio by:

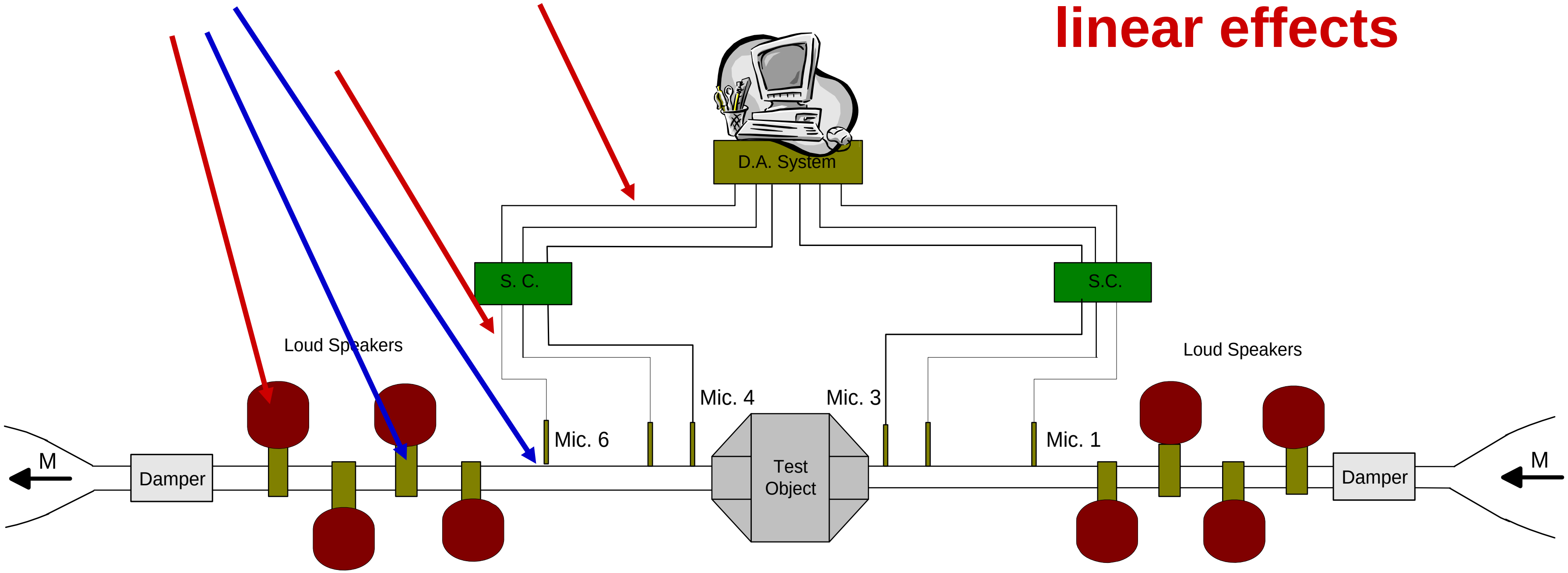
- Increasing level of input signal
- Using multiple loudspekers
- Using high level loudspekers
- Concentrating signal energy to narrow frequency bands

Flow Noise Supression

Use reference signal and ”correlation” techniques:

Possible reference signal locations

Risk for non-linear effects



Signal enhancement techniques

1) Frequency domain averaging (**FDA**). Welch's technique.
2) Synchronised time domain averaging (**STDA**). Requires deterministic signal + reference (trig) signal.
3) Cross-spectrum based frequency domain averaging (**CSFDA**). Requires noise free reference.

Auto-spectrum estimation
$$\hat{G}_{xx}(\omega)=\frac{\hat{G}^{*}_{rx}(\omega)\hat{G}_{rx}(\omega)}{\hat{G}_{rr}(\omega)}$$

Averaging
$$\hat{G}_{\gamma x}(\omega)=\frac{1}{N}\sum_{i=1}^{N}G^{i}_{\gamma x}(\omega)$$

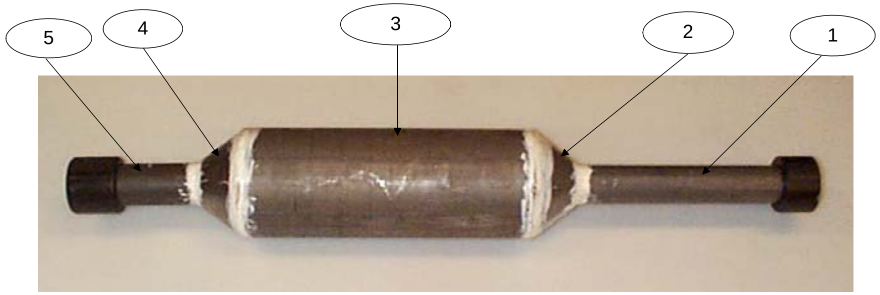
An improvement in SNR by a factor of *N* can be expected or $10\cdot Log(N)$ dB

Test objects for two-port measurements

Three configurations tested

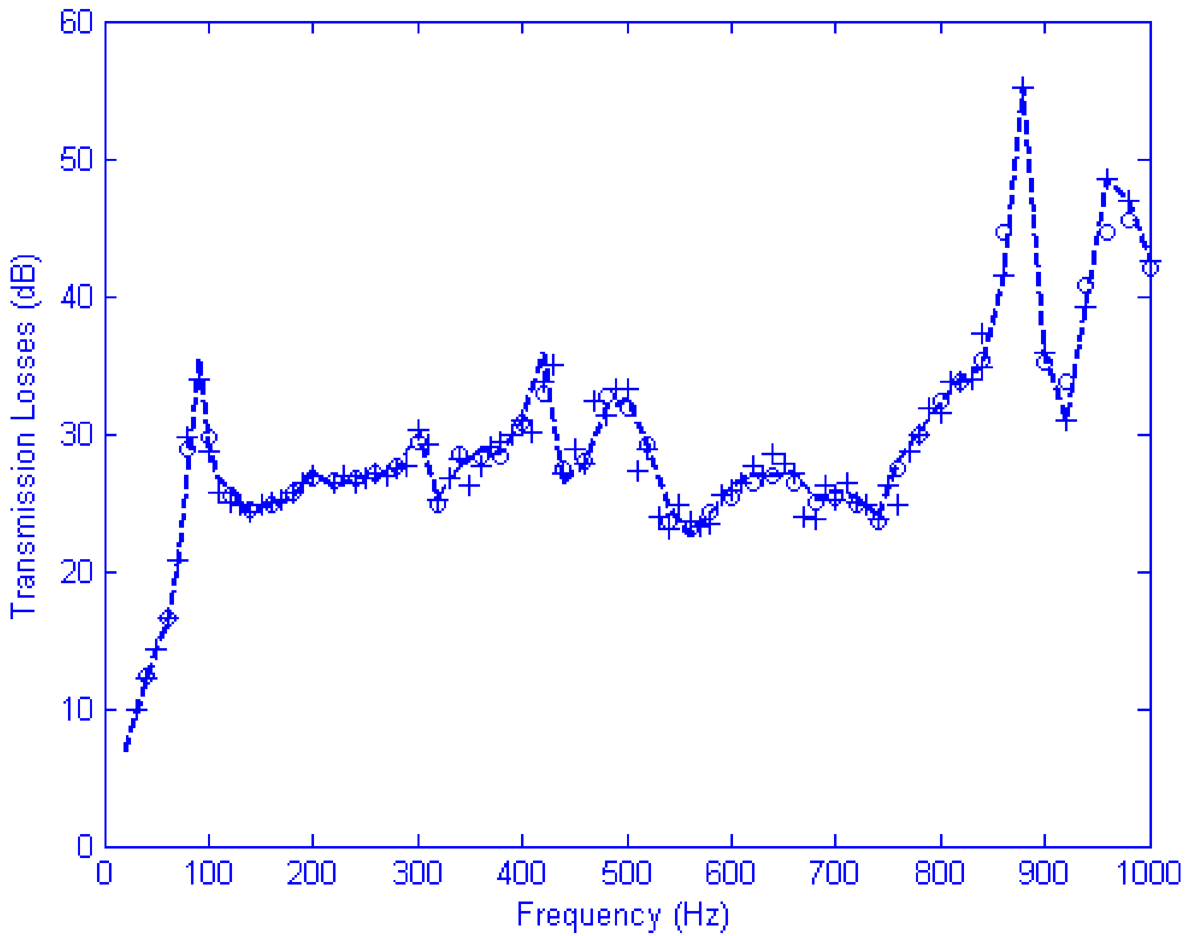
1) Straight hard-walled pipe. Theoretical solution available.

2) Expansion chamber



3) Automotive muffler

Commercial muffler



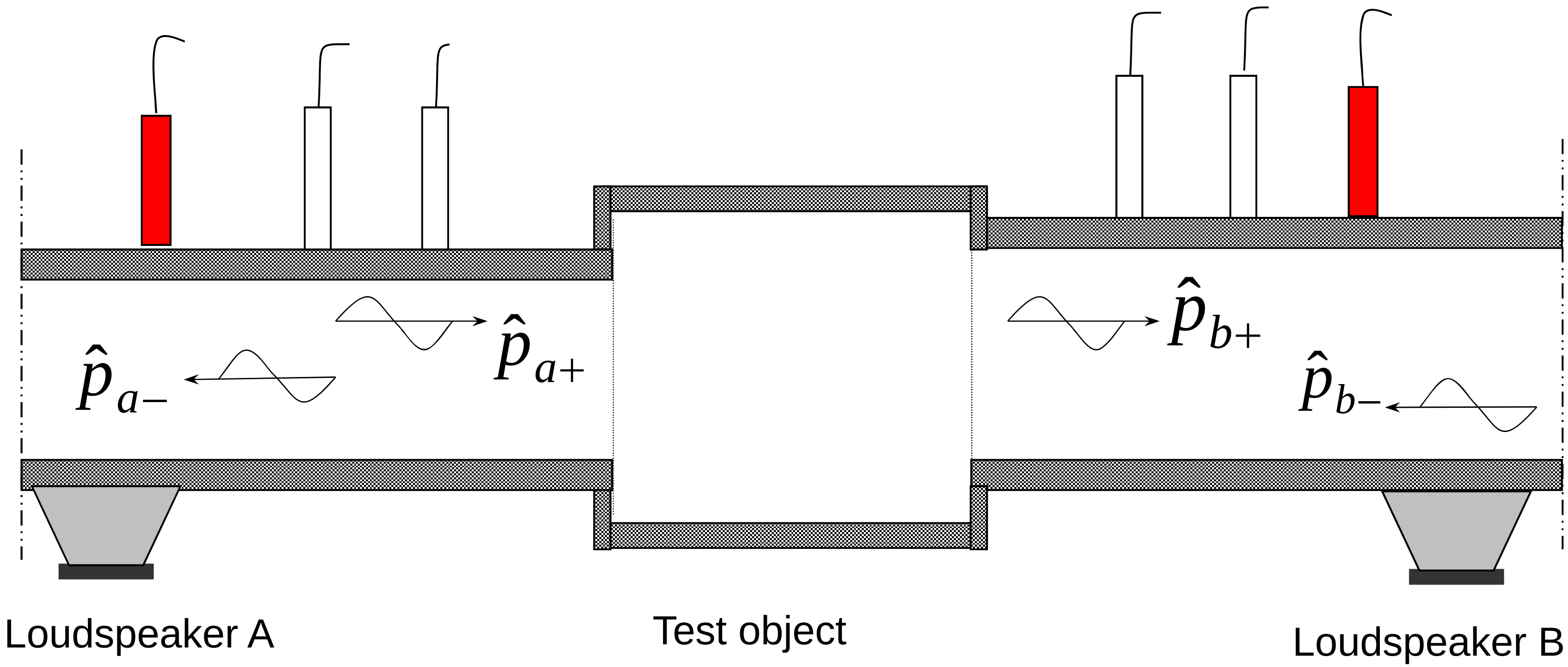
Transmission loss at M=0.26.

----, random excitation 10000 averages (CSFDA)

++++, sawtooth excitation 10000 averages (STDA)

oooo, stepped sine excitation 400 averages (CSFDA)

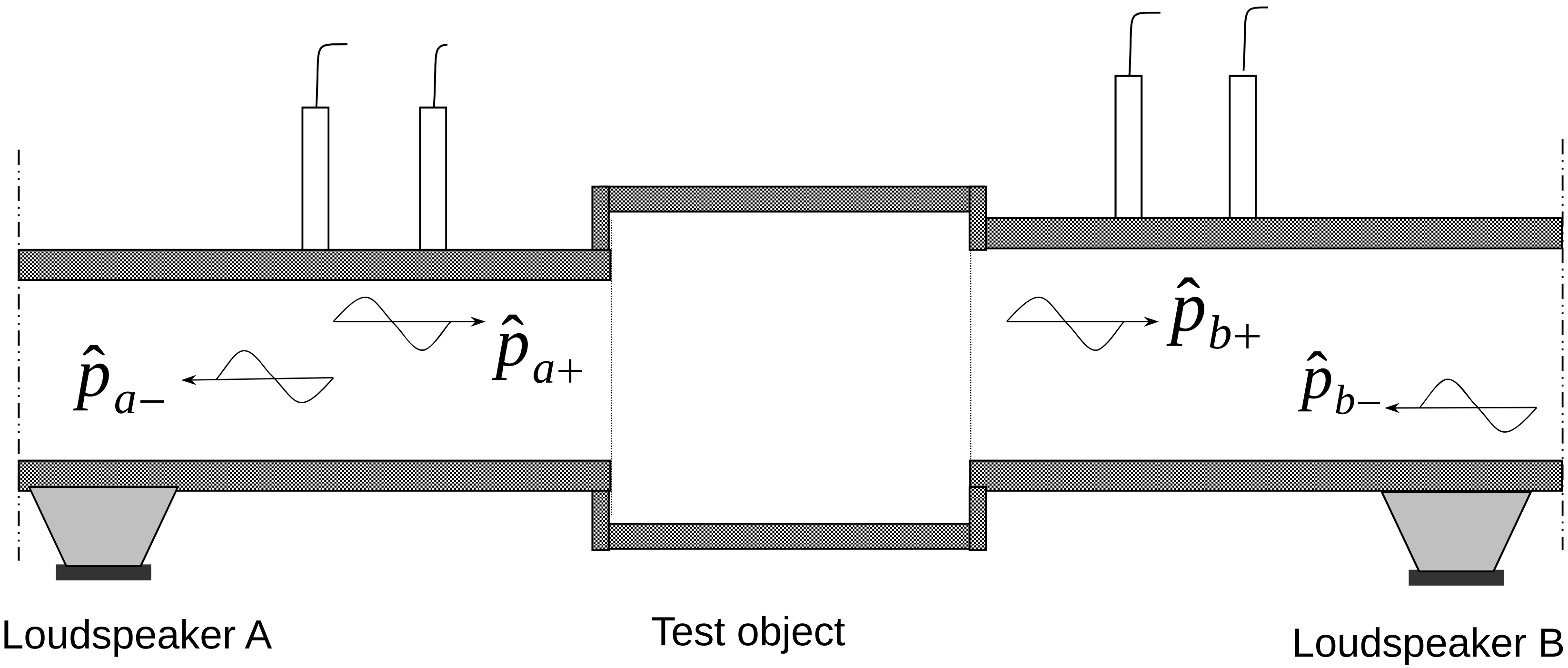
Two-port measurements with over-determination



$$\begin{pmatrix} p_a \\ q_a \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b \\ q_b \end{pmatrix}$$

Add additional microphones

Two-port measurements with over-determination



$$\begin{pmatrix} p_a \\ q_a \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b \\ q_b \end{pmatrix}$$

Use: A, B, A+B, A-B, B-A.....
Does not give additional independent equations

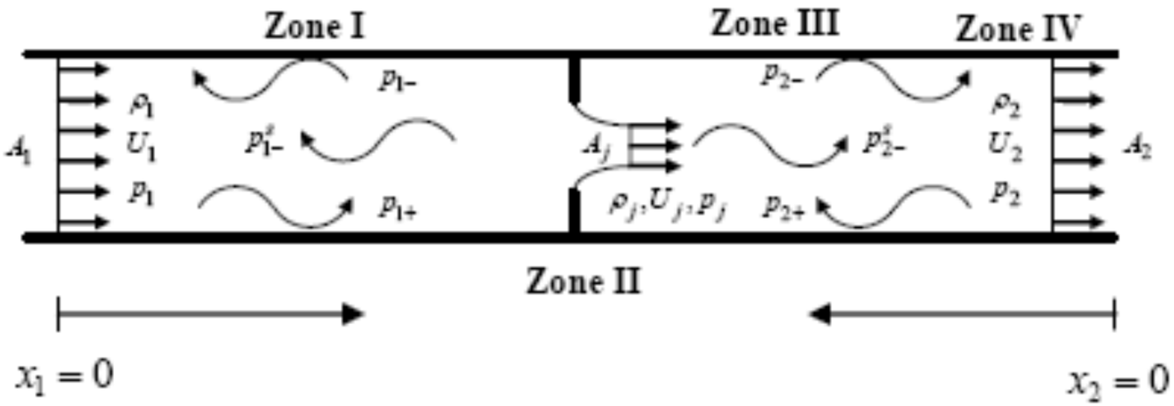
Results and discussion

Measurements made for single diaphragm orifice

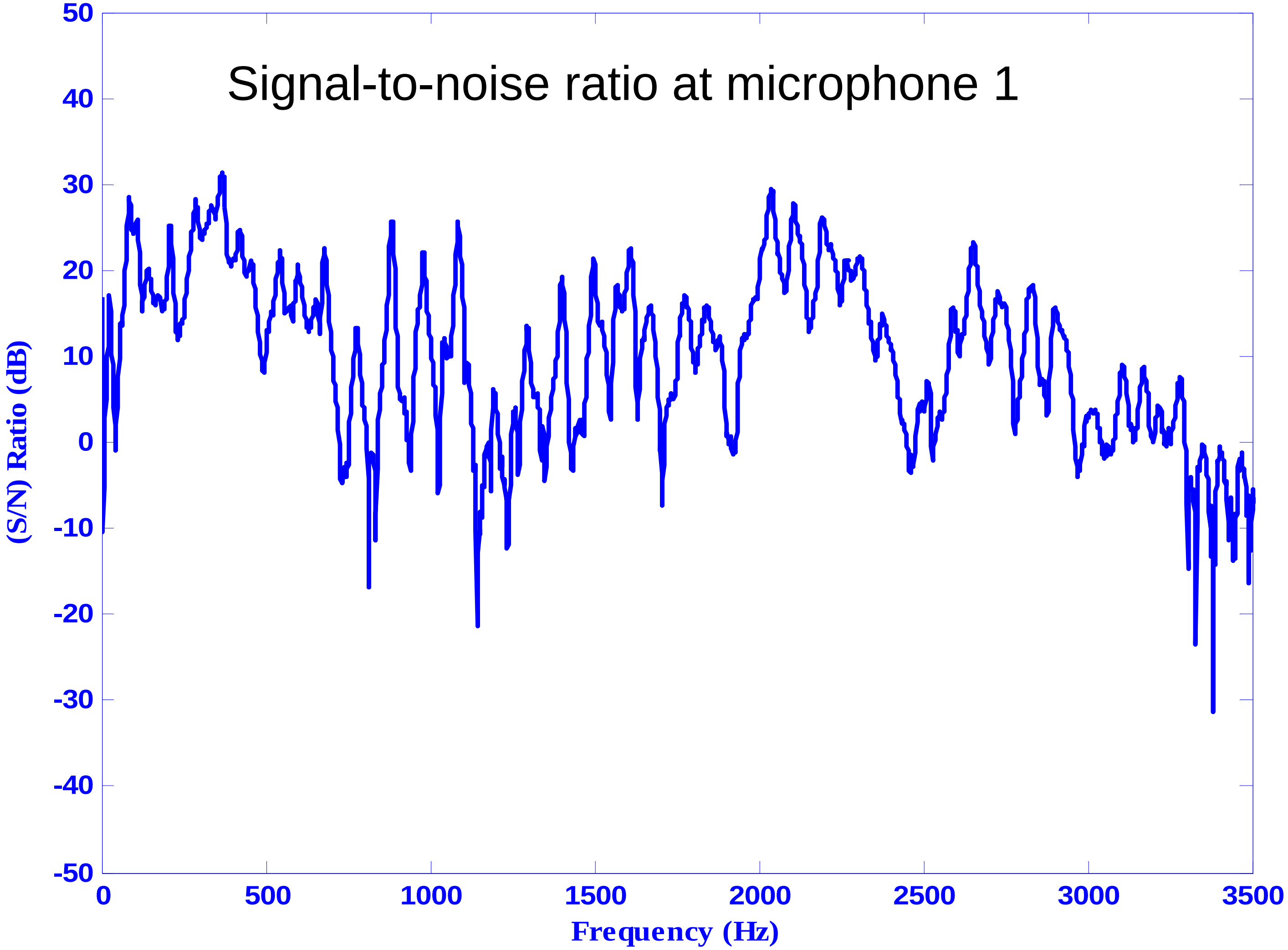
5 different test cases

Comparison to FEM prediction (FEMLAB)

Low signal-to-noise ratio



Results and discussion



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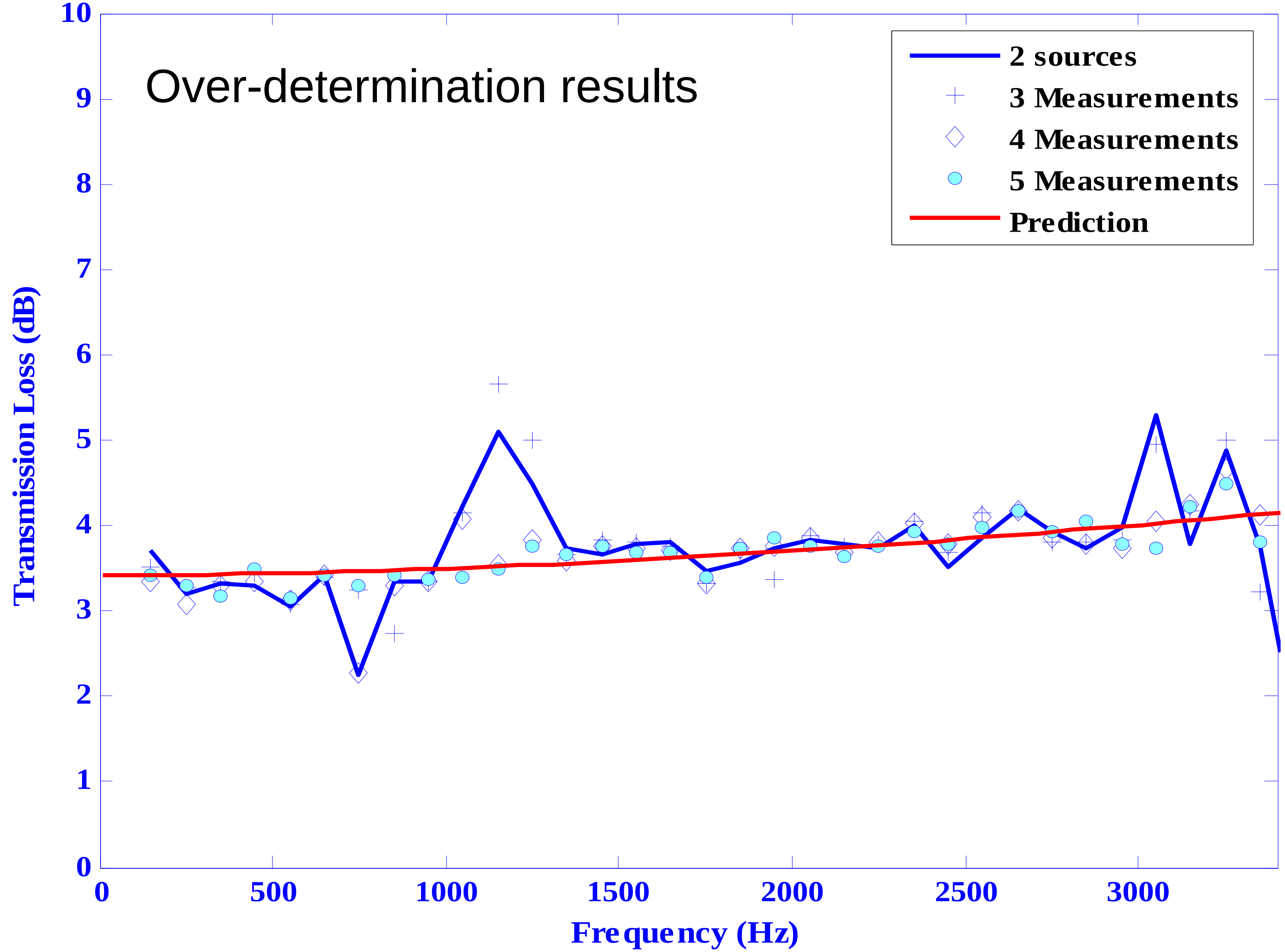
Results and discussion



Aero-Engine Liners

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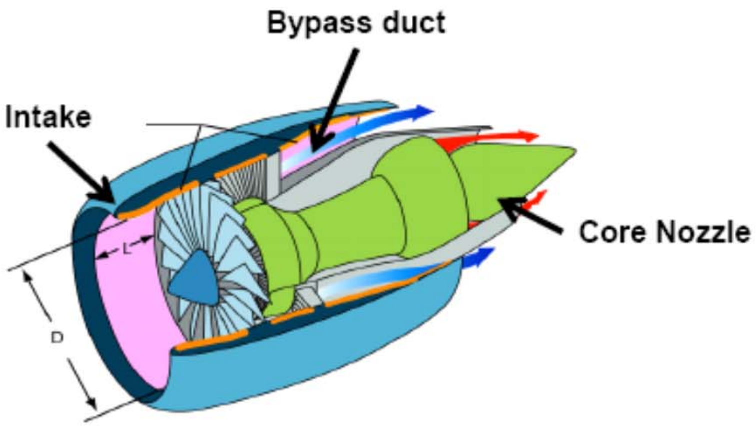
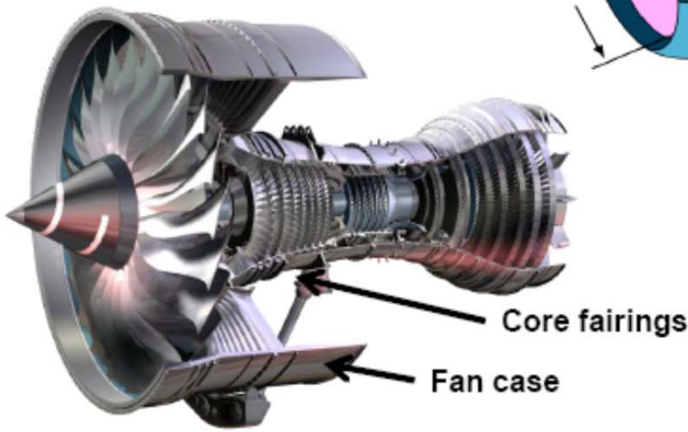
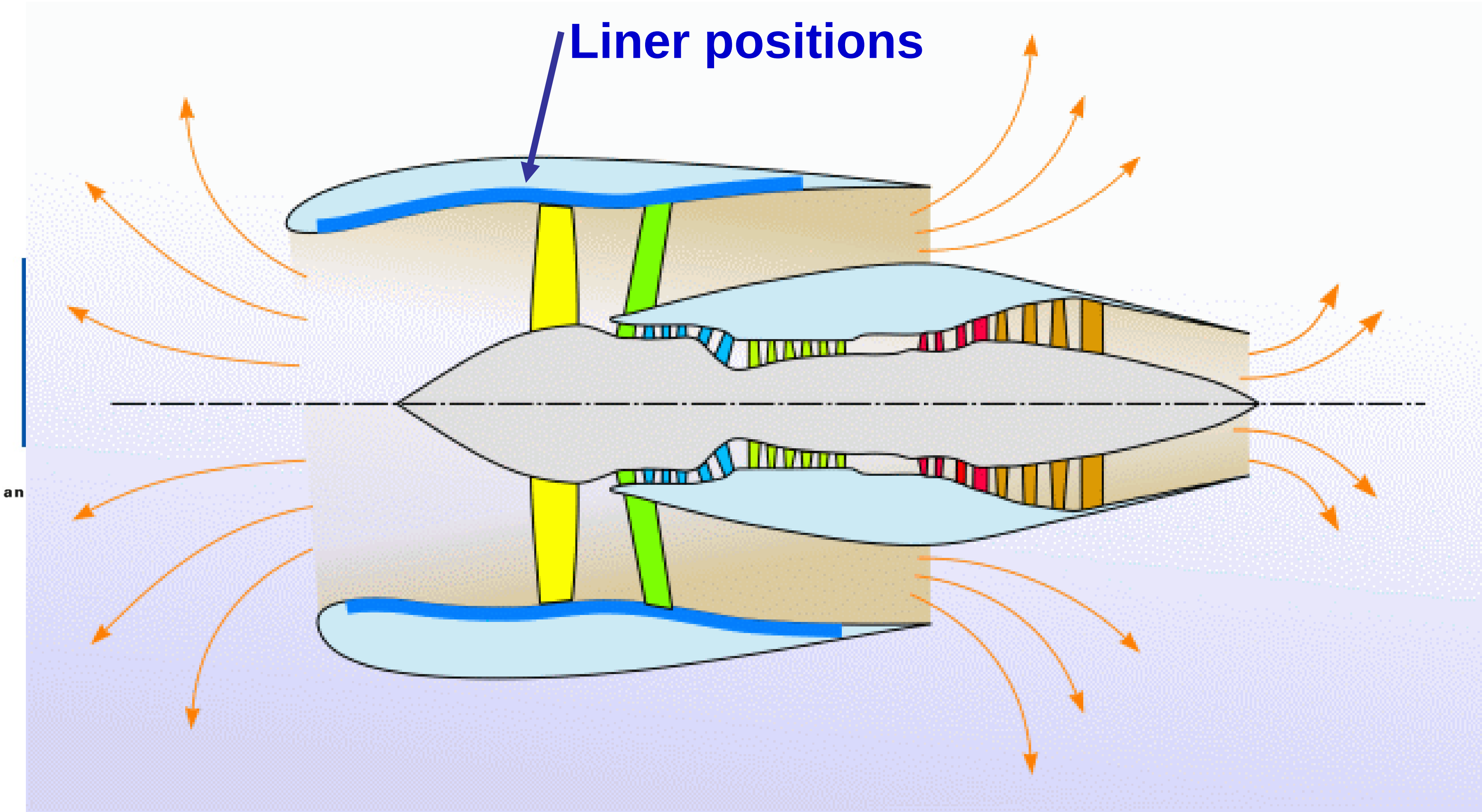




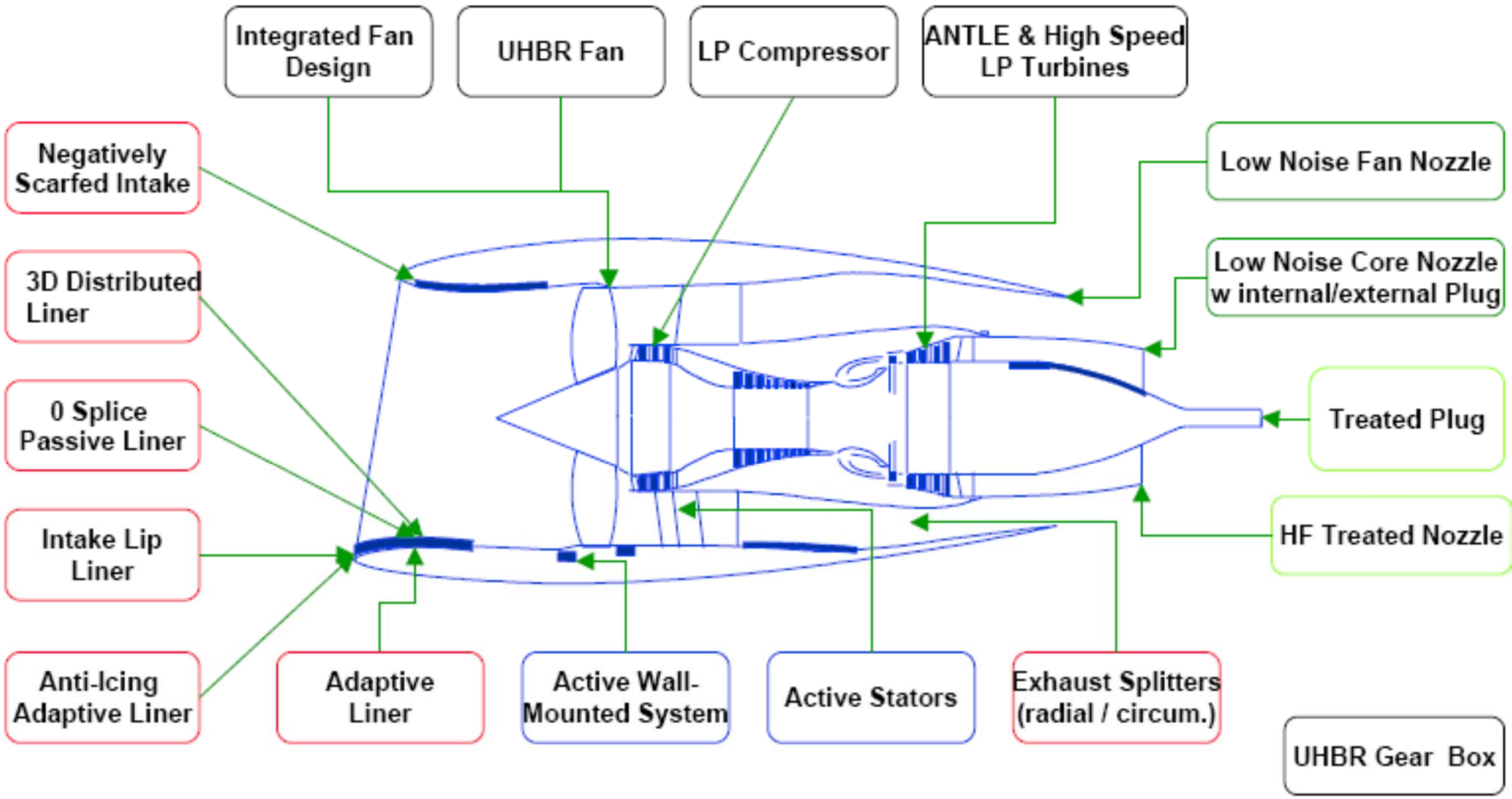
Rolls-Royce

Aircraft Engine Liners

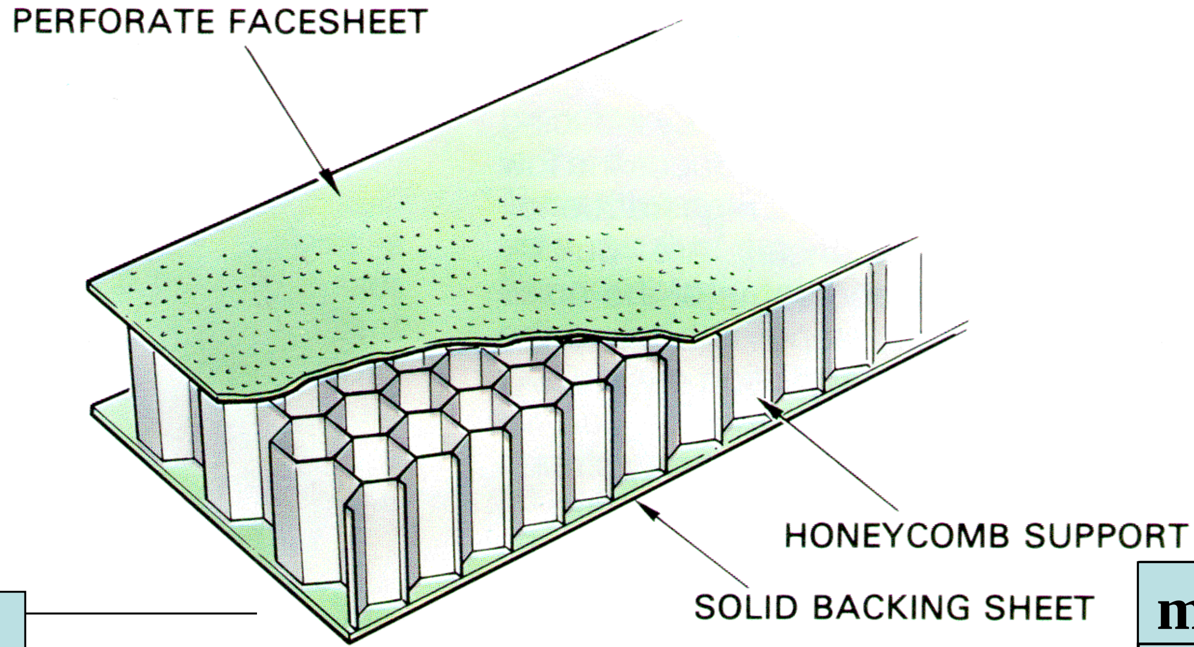


SILENCE(R)

How to reach Objectives



Aircraft Engine Liner



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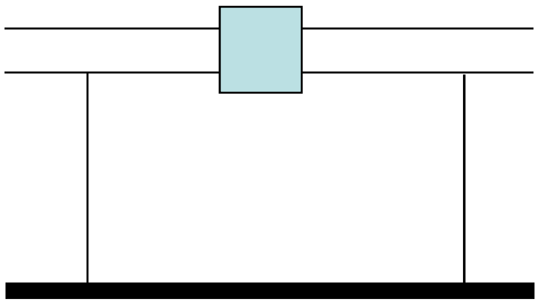
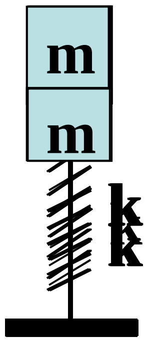


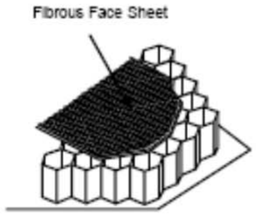
Helmholtz Resonator

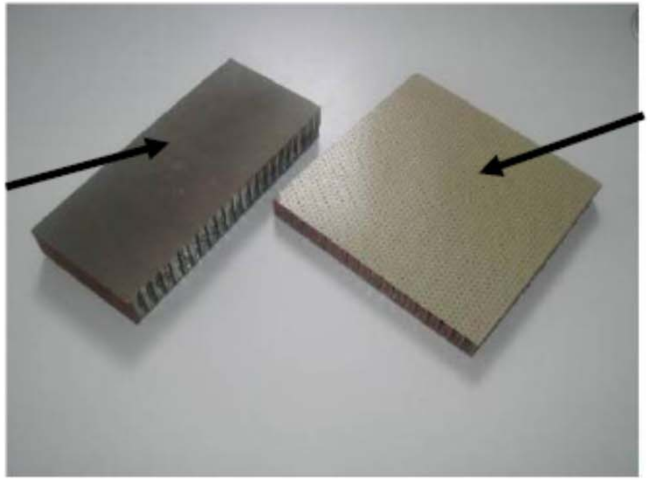


Liner Types in common use

Single Layer Linear

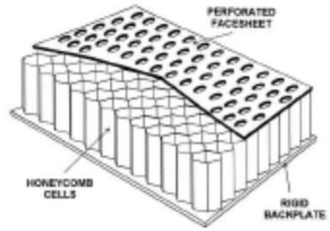
Wire mesh on perforate sheet + honeycomb + hardwall backing sheet



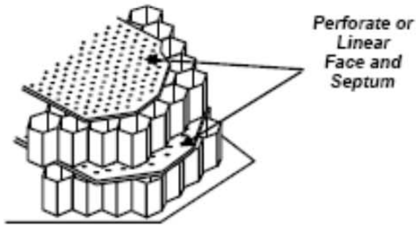


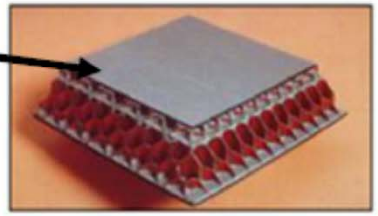
Single Layer Perforate

Perforate sheet + honeycomb + hardwall backing sheet



Double layer liners





Perforated sheets

- ***Punched Aluminium***
- ***Mechanically Drilled Carbon Fibre Composites***
- ***Laser Drilled***
- ***Injection Moulded***
- ***Microperforates***

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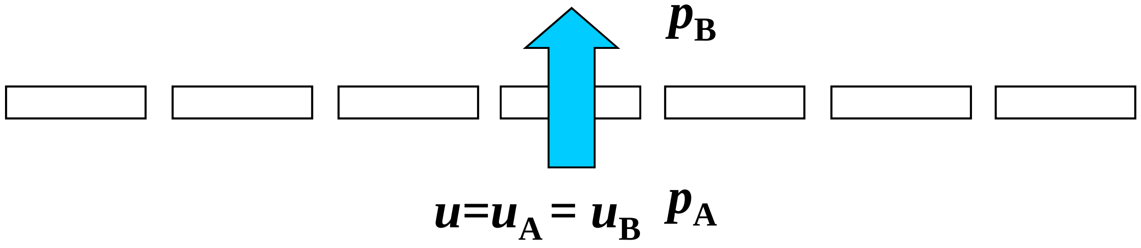
Rolls-Royce

Normalised Acoustic Impedance



$$Z = \frac{\Delta p}{\rho_0 c u} = \frac{\left(1 - \sigma^2 C_D^2\right)}{2 c C_D^2} \cdot |u| + jkl$$

This neglects linear (viscous) losses, flow effects etc.

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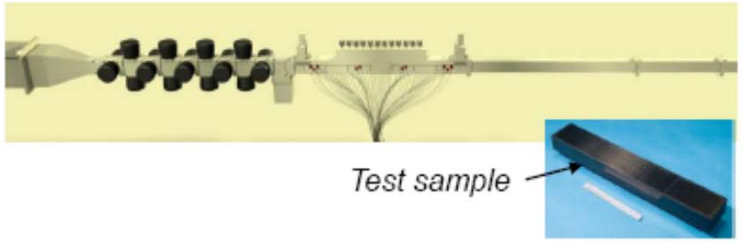
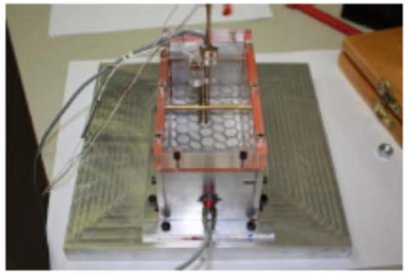
Liner Impedance Measurements

DC Flow Resistance

NASA Grazing Flow Impedance Tube



Portable Acoustic Liner Meter



NLR In-Situ Impedance Measurements

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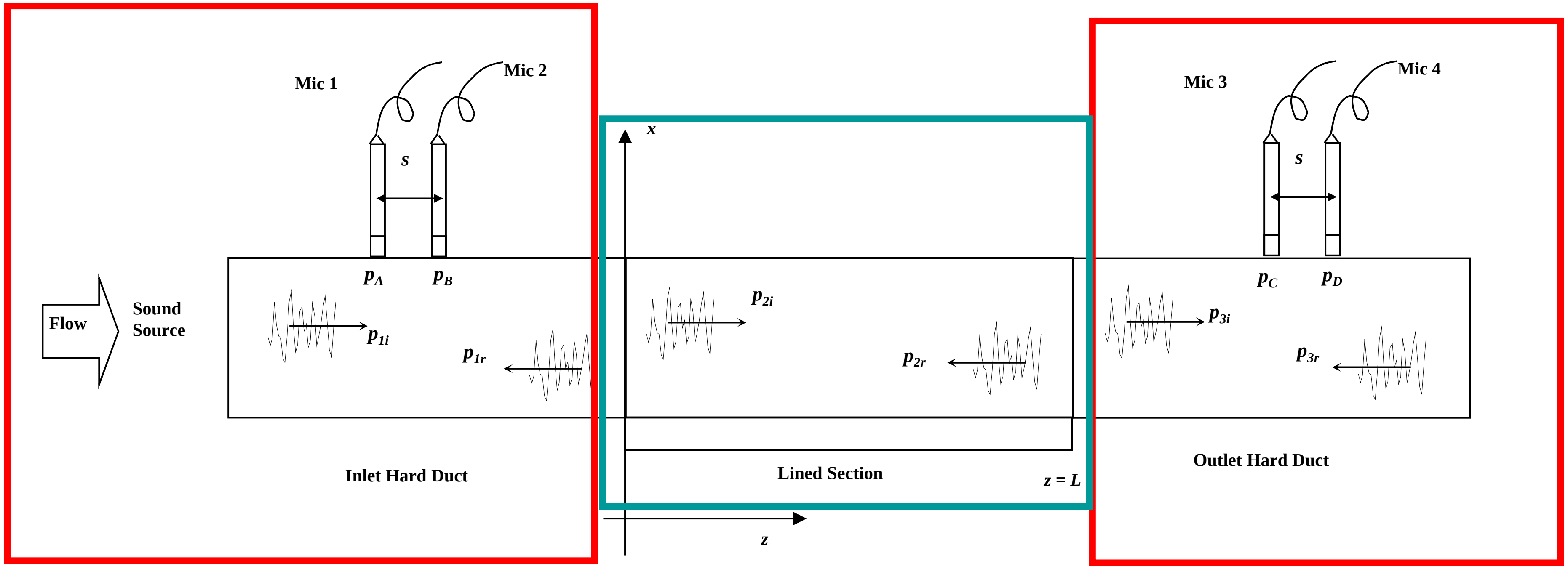
Grazing flow impedance "eduction" – Inverse determination of impedance boundary condition



Model for sound propagation in lined section

Plane wave decomposition

Models for perforate impedance

Elnady, T.,
Modelling and characterization of perforates in lined ducts and mufflers, *PhD Thesis*, KTH, Stockholm, Sweden, 2004.

Normalised resistance

$$\theta = \mathrm{Re}\left(\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')} + \frac{\delta_{re}}{F(\mu)} f_{\mathrm{int}}\right]\right) + \frac{1}{\sigma}\left[1 - \frac{2J_1(kd)}{kd}\right] + \left(\frac{1-\sigma^2}{\sigma^2 C_D^2}\right)\cdot\frac{1}{2c}|v_n| + \frac{0.5}{\sigma}\cdot M$$

Normalised reactance

Nonlinear terms

Flow terms

$$\mathrm{X} = \mathrm{Im}\left(\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')} + \frac{0.5d}{F(\mu)} f_{\mathrm{int}}\right]\right) - \left(\frac{1-\sigma^2}{\sigma^2 C_D^2}\right)\cdot\frac{1}{2c}\cdot\frac{|v_n|}{3} - \frac{0.3}{\sigma}\cdot M$$

$$K = \sqrt{-\frac{j\omega}{\nu}}$$

$$\delta_{re} = 0.2d + 200d^2 + 16000d^3$$

$$F(\mu) = 1 - \frac{4J_1(Kd/2)}{Kd\cdot J_0(Kd/2)}$$

$$f_{\mathrm{int}} = 1 - 1.47\sqrt{\sigma} + 0.47\sqrt{\sigma^3}$$

The acoustic properties can become non-linear at high acoustic excitation levels

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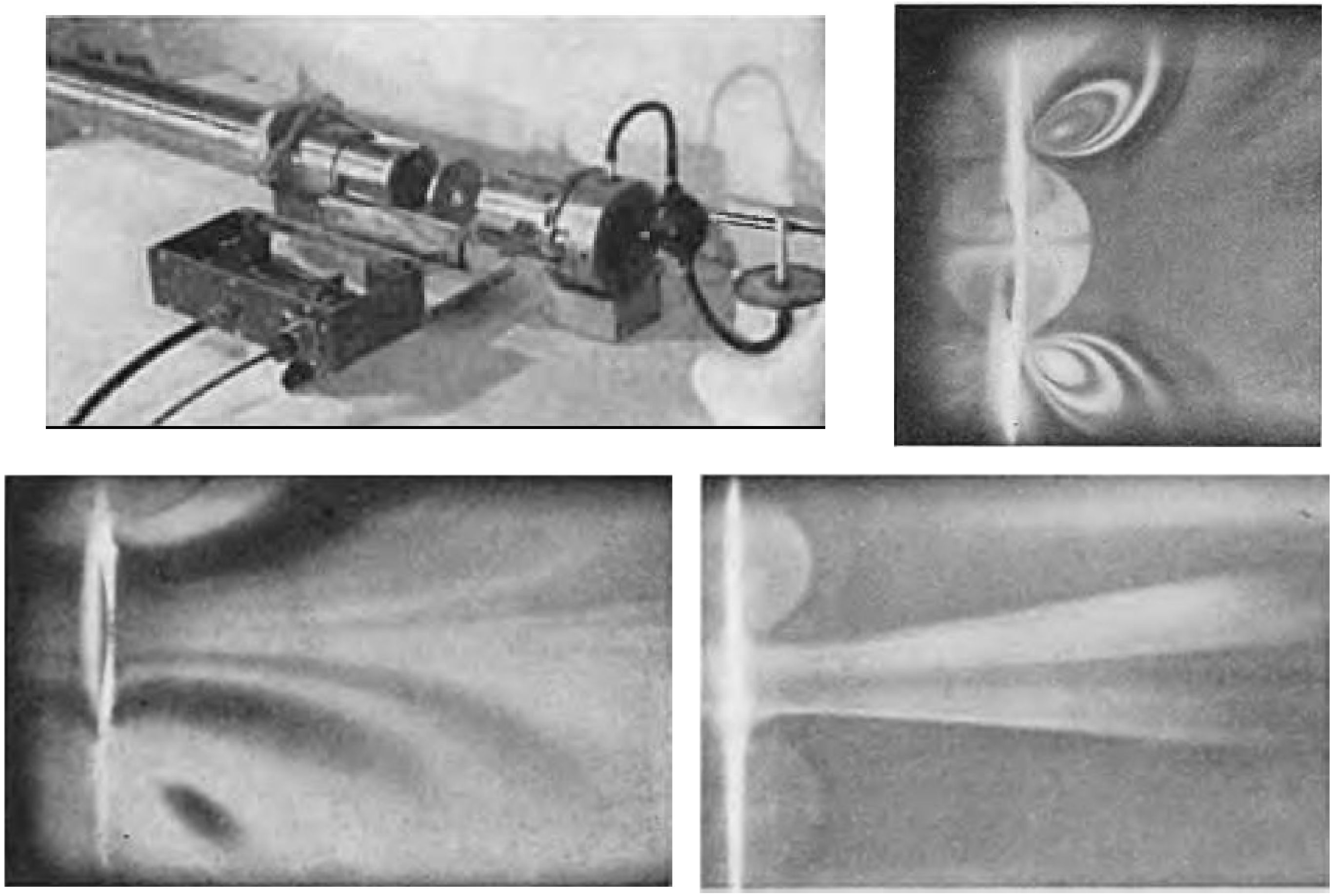

Ingård and Labate JASA, 1950

The acoustic properties can become non-linear at high acoustic excitation levels

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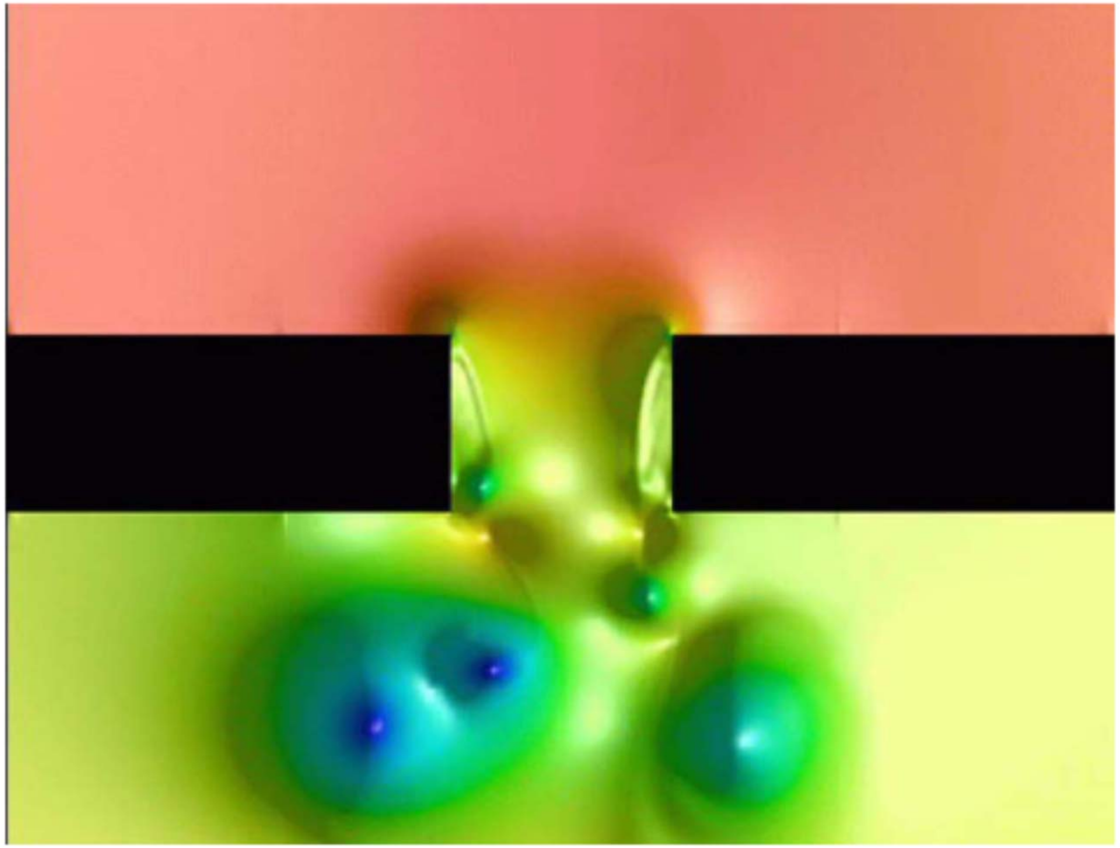
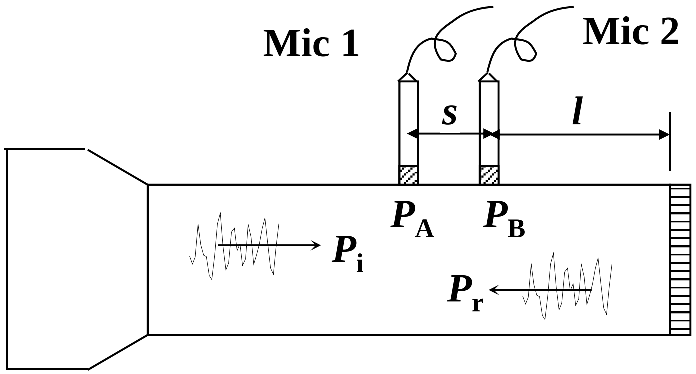


Fig. 28. Vortex shedding at a 90° slit.

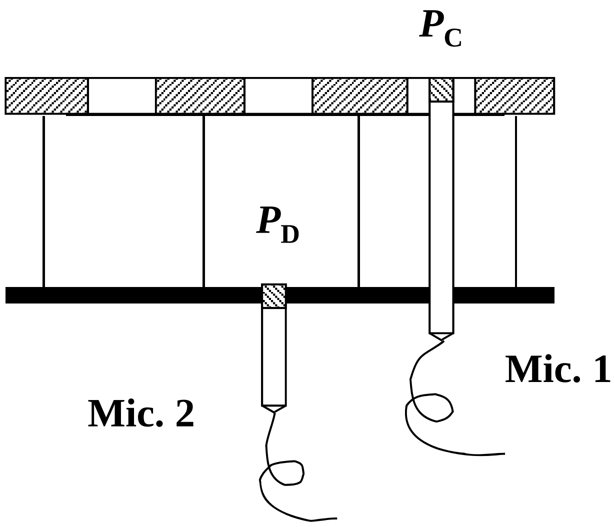
Tam et. al. JSV 284, 205

Normal Incidence Impedance Measurement Techniques

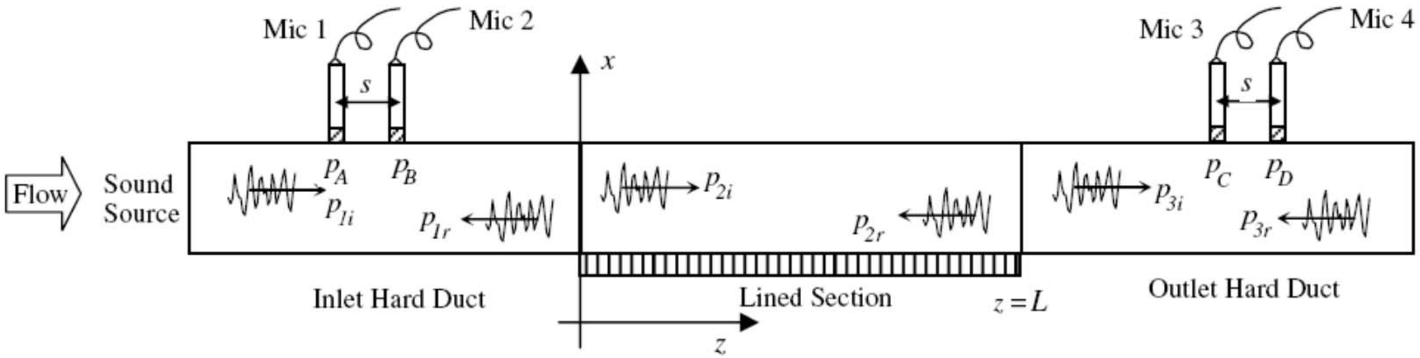
In-duct, two microphone technique:

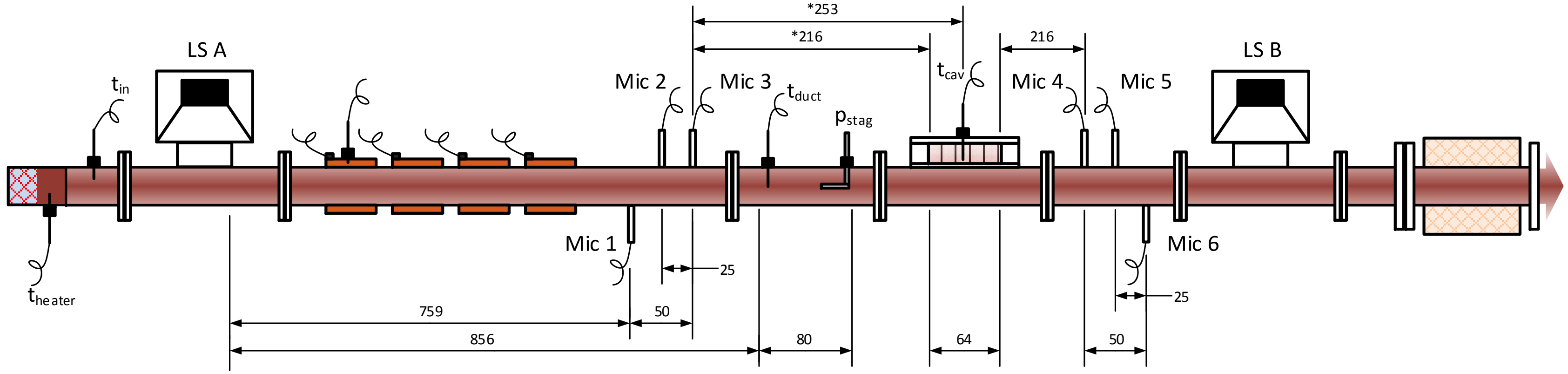


In-situ two-microphone technique:



Grazing Flow Impedance Measurements





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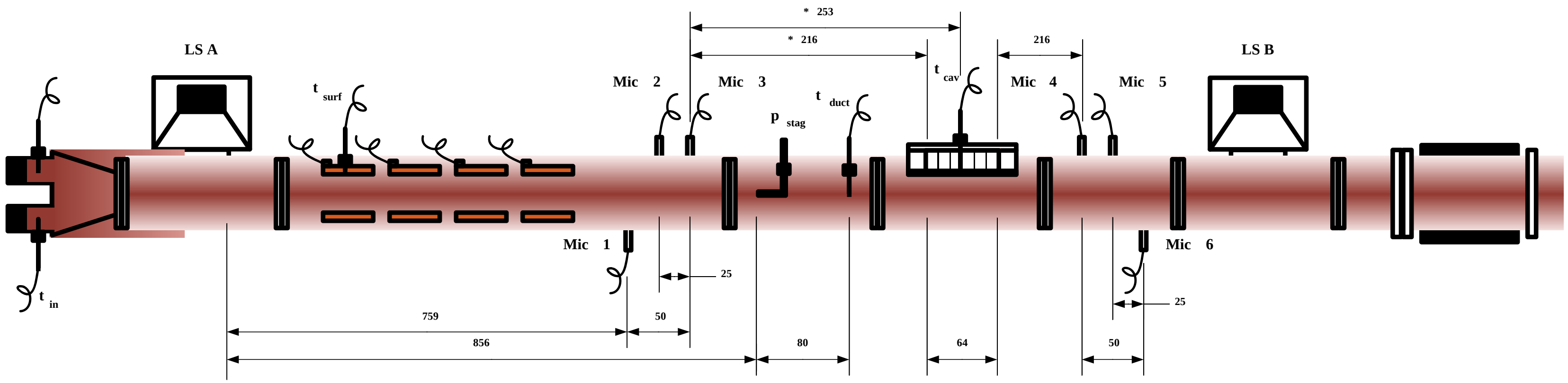
High flow speed test rig

High temperature test rig



Impedance Eduction

- **W. Watson, M. Jones, T. Parrott, S. Tanner (NASA Langely Research Center)**

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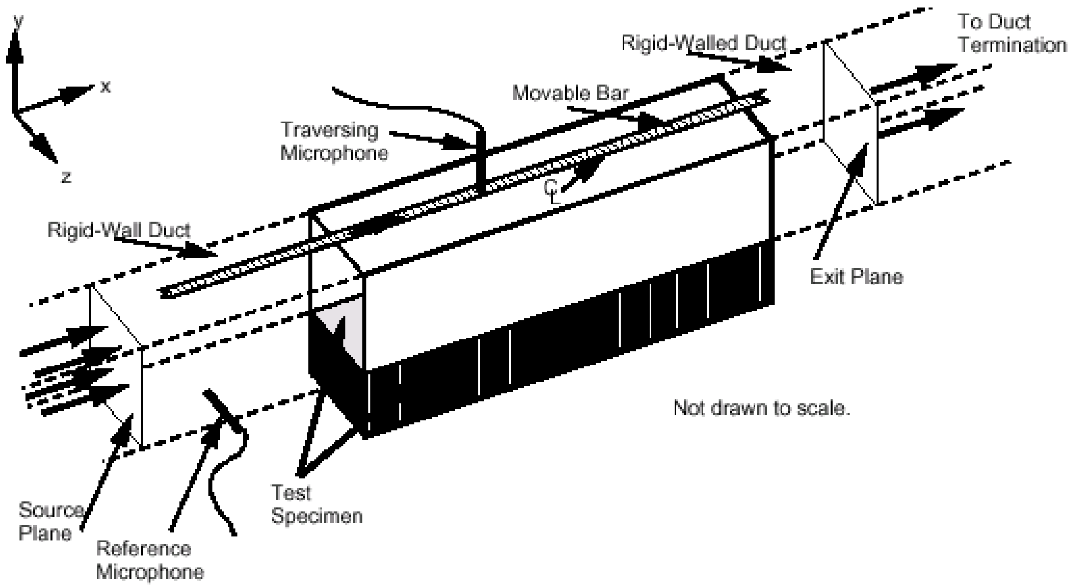

Computation Domain

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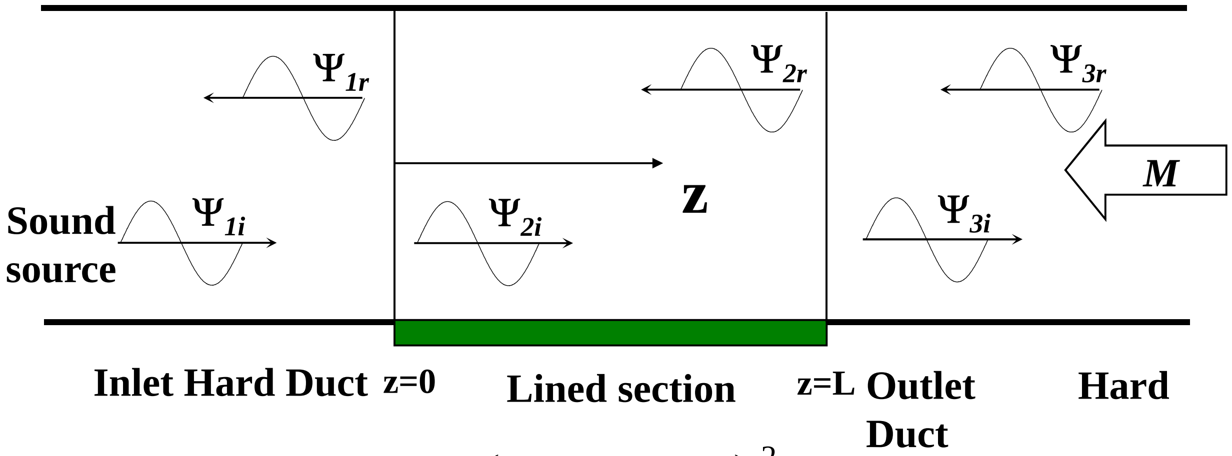
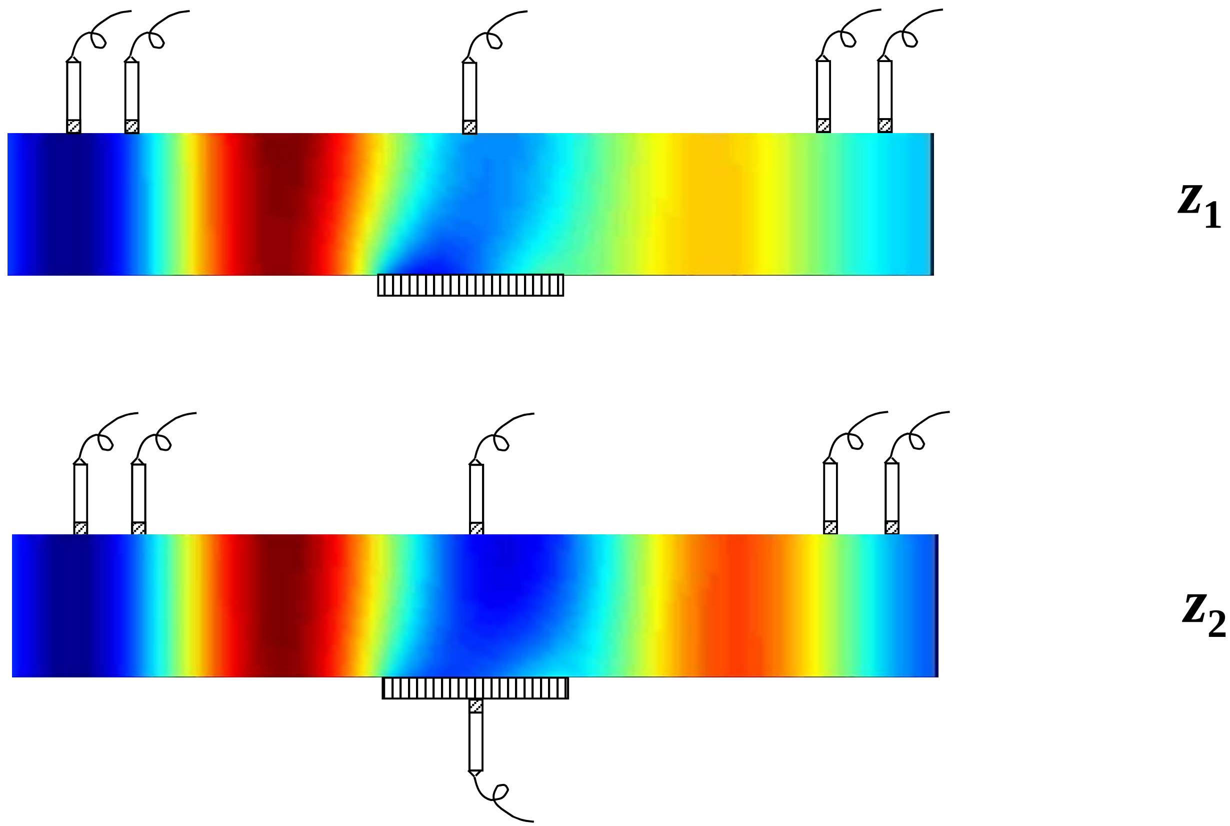


$$\nabla^2\Psi_2 - \left(jk + M\frac{\partial}{\partial z} \right)^2 \Psi_2 = 0$$

$$\Psi_2(x,y,z) = \sum_{q=1}^{Q} b^{(q)} \cdot \psi_2^{(q)}(x,y) \cdot e^{-jk_{z2}^{(q)} \cdot z}$$

Experimental determination of grazing flow impedance



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Eduction procedures

Convected Hemlholtz Equation Mode matching
Straight Forward Method
Pridmore Brown Equation

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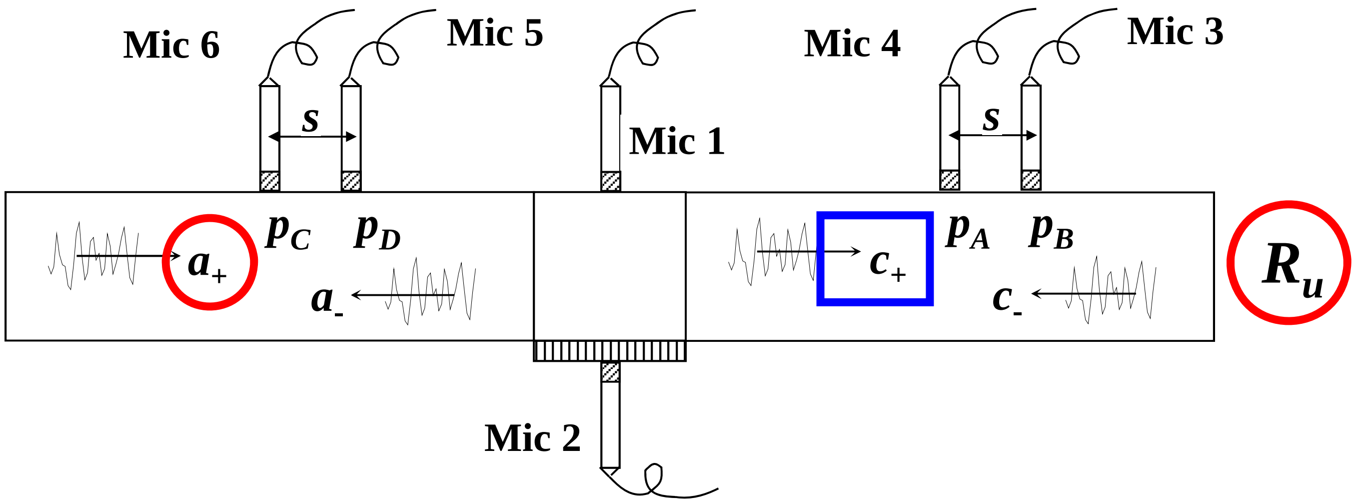

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Convected Hemlholtz Equation Mode matching

Analytical solution for sound field in lined section with known (guessed) impedance
Mode matching between lined and hard sections
Optimization, minimizing the difference between measured and calculated pressures at the microphone positions.

KTH, Straight forward method

Assume only one propagating mode in the lined section
Measure the scattering matrix of the lined section
Calculate transverse wave numbers from the scattering matrix
Calculate the axial wave number assuming no reflection
Calculate the impedance

Comparison of impedance eduction results using different methods and test rigs. L. Zhou, H. Bodén, C. Lahiri, F. Bake, L. Enghardt and T. Elnady. AIAA/CEAS Aeroacoutics Conference, Atlanta 2014.

DLR, LINUS method

Measure the scattering matrix of the lined section
Calculate transverse wave numbers from the scattering matrix
Calculate the axial wave number assuming no reflection
Integrate the pressure and its gradient over the cross section
Calculate the impedance

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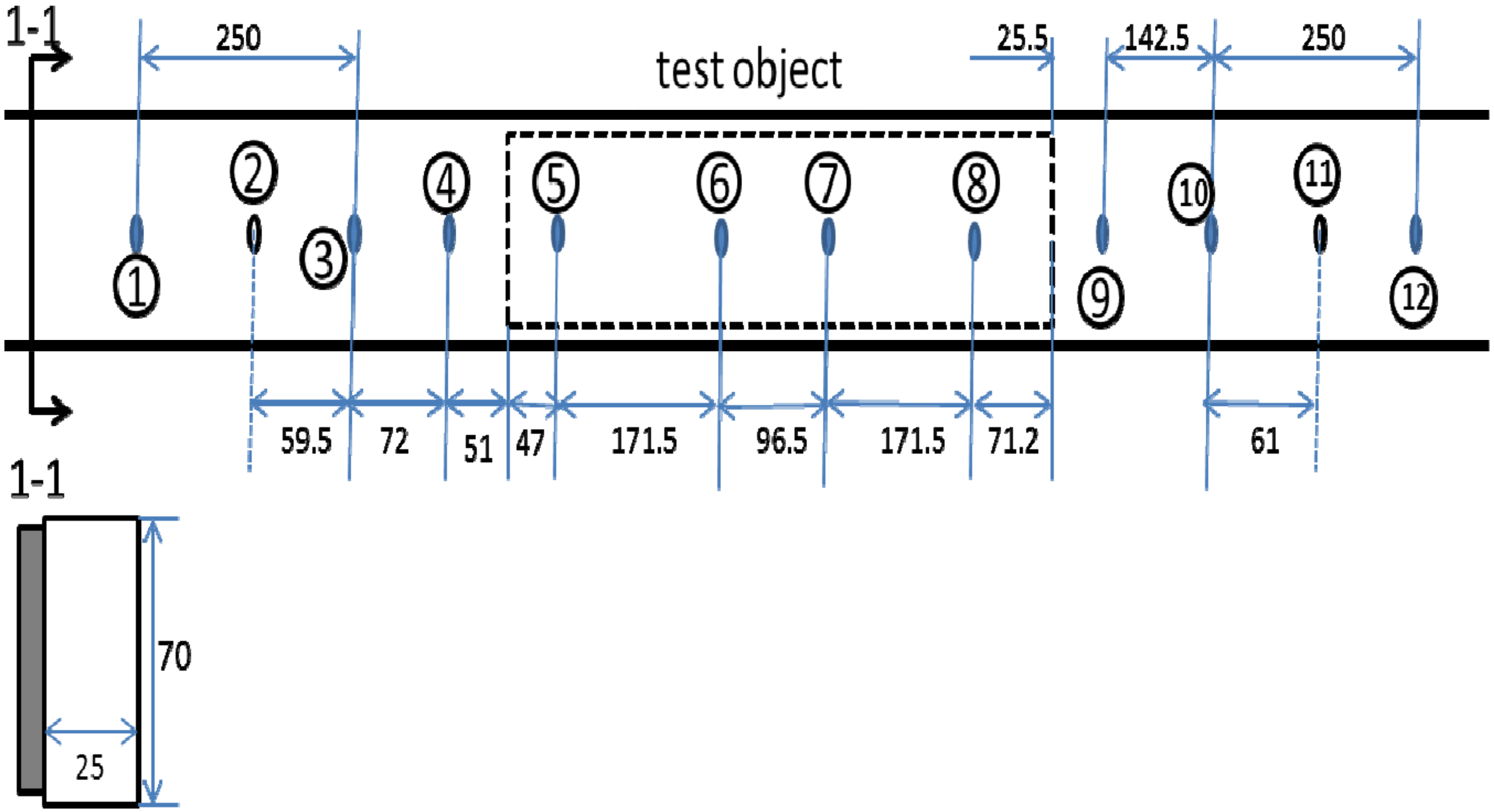
Comparison of impedance eduction results using different methods and test rigs. L. Zhou, H. Bodén, C. Lahiri, F. Bake, L. Enghardt and T. Elnady. AIAA/CEAS Aeroacoutics Conference, Atlanta 2014.

Microphone positions



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Impedance Eduction Results

Black – only microphones outside the lined section,

Red – including three microphones in the lined section

Convected Helmholtz equation, mode matching method

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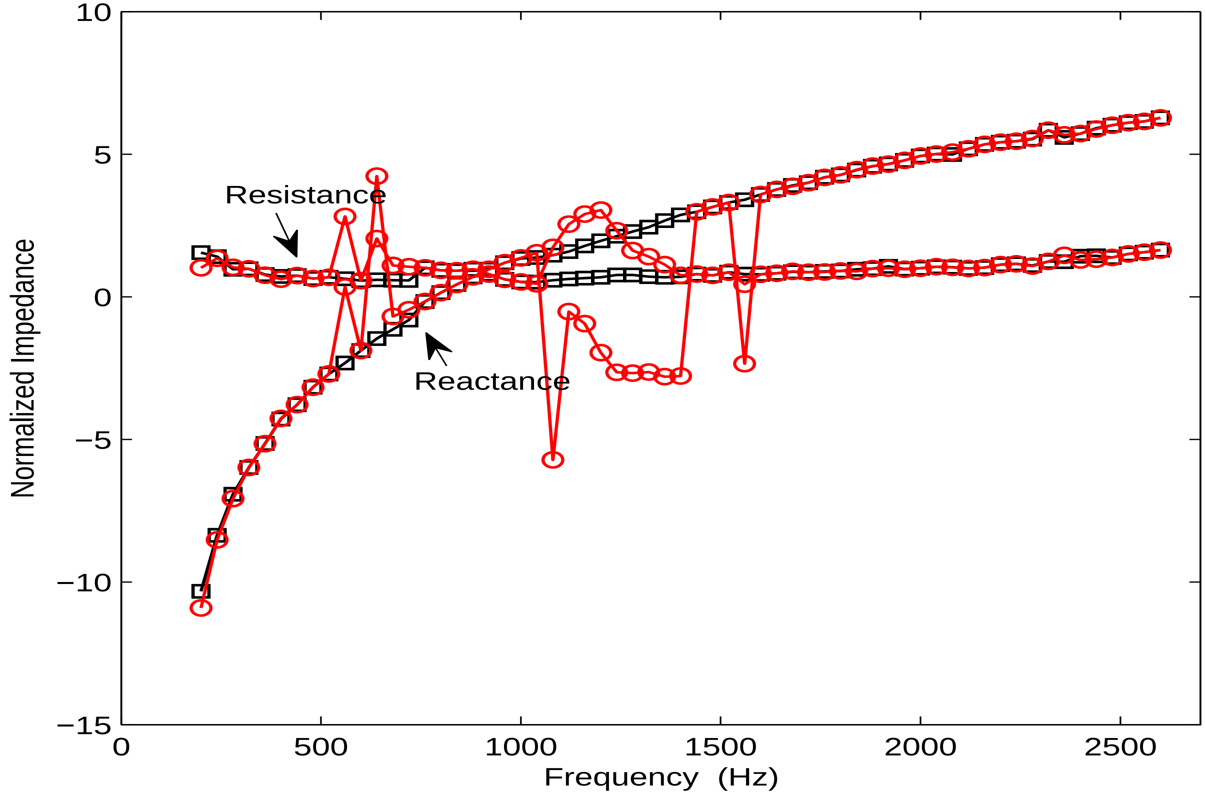


M = 0

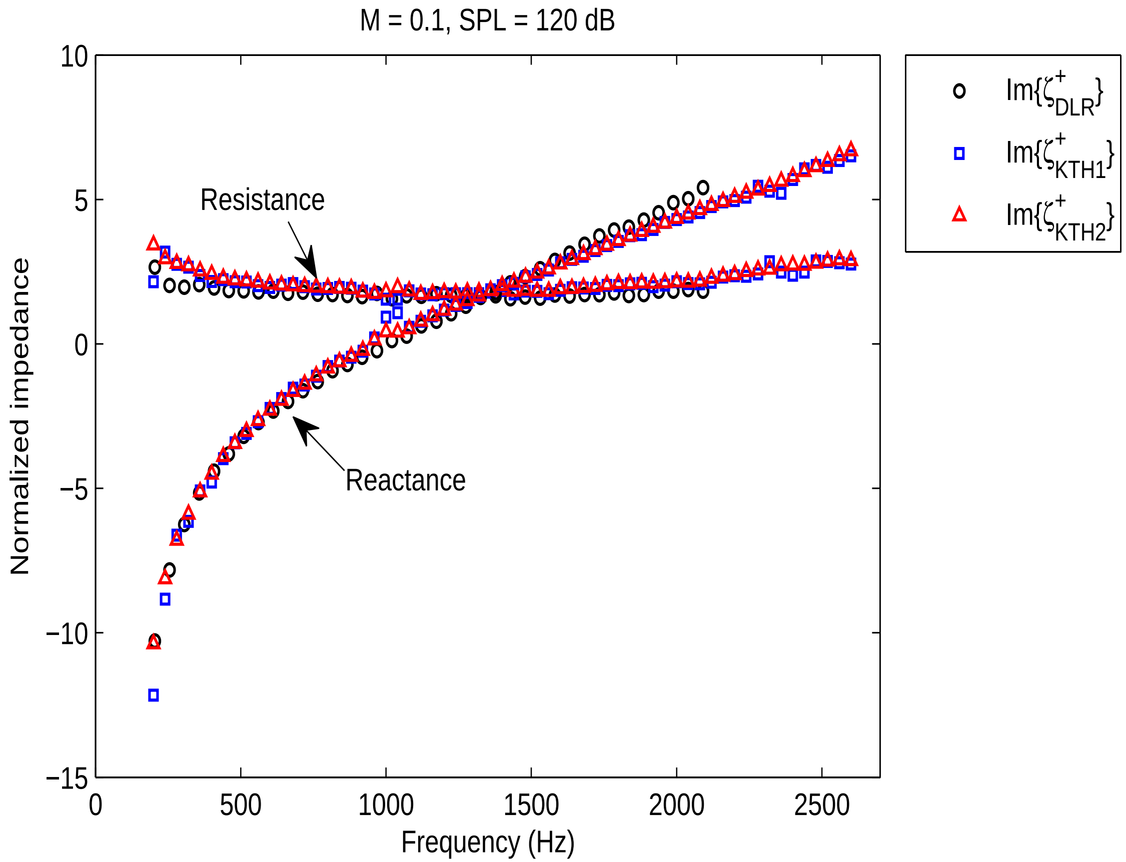
120 dB

Impedance Eduction Results

Blue – Convected Helmholtz Equation mode matching method

Red – Strigth forward method

Black – DLR Linus method



M = 0.1
120 dB

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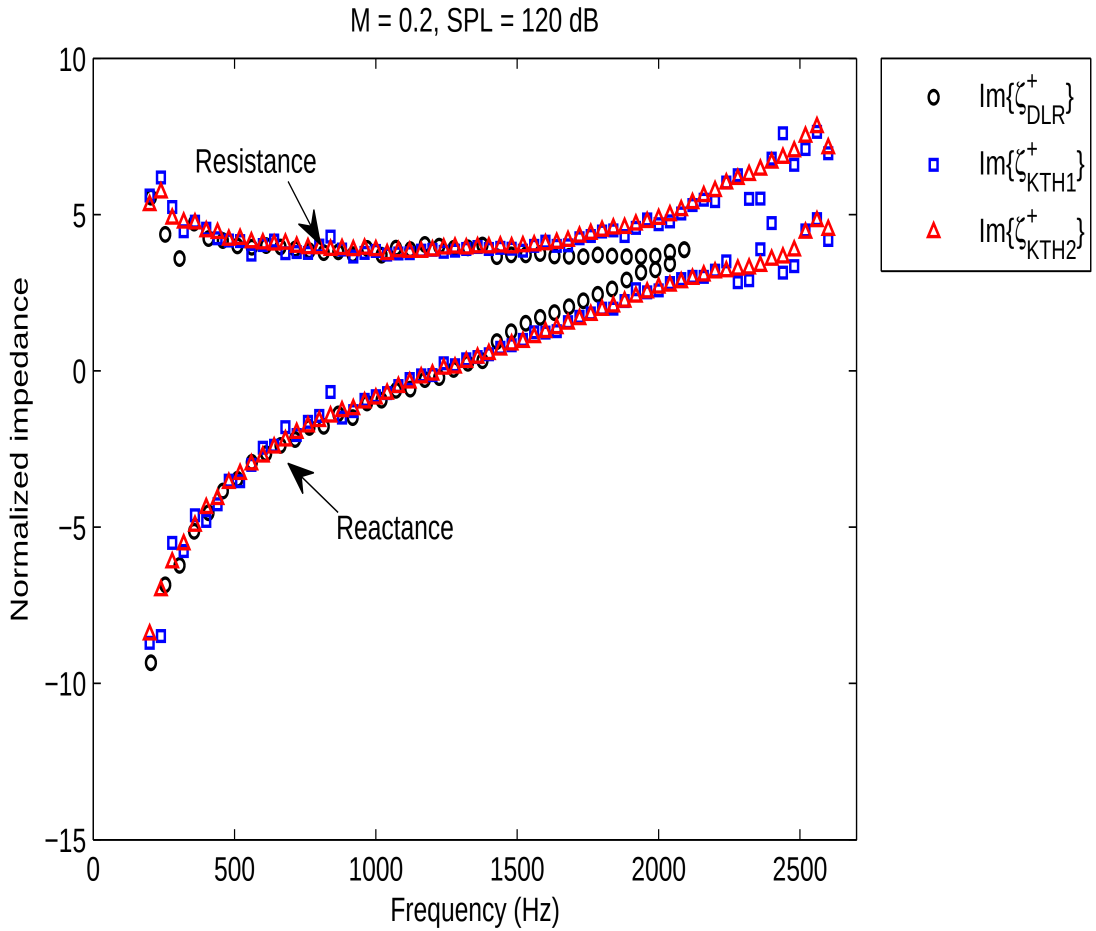
Impedance Eduction Results

Blue – Convected Helmholtz Equation mode matching method

Red – Strigth forward method

Black – DLR Linus method



M = 0.2
120 dB

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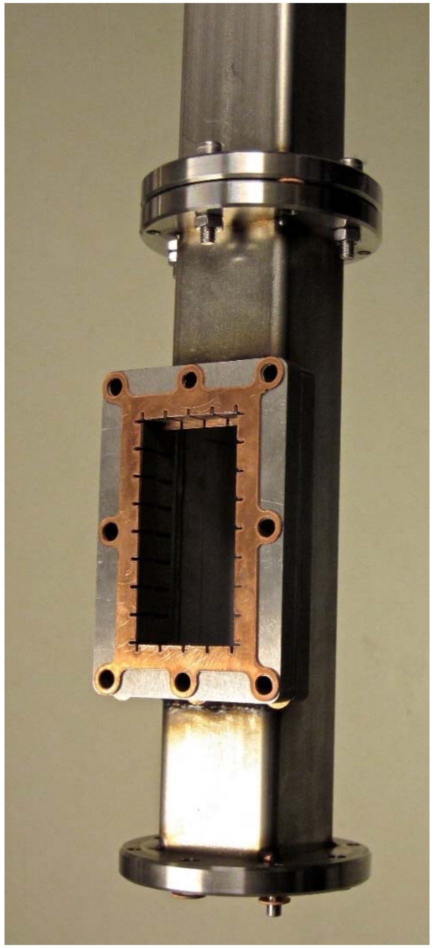
Hot stream liner impedance measurements



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Liner Samples for Hot Stream Liner Testing

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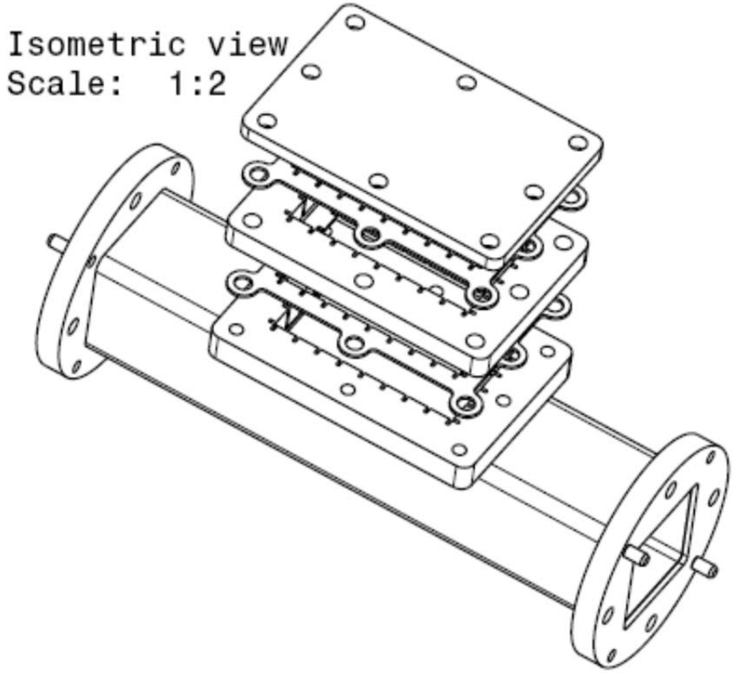

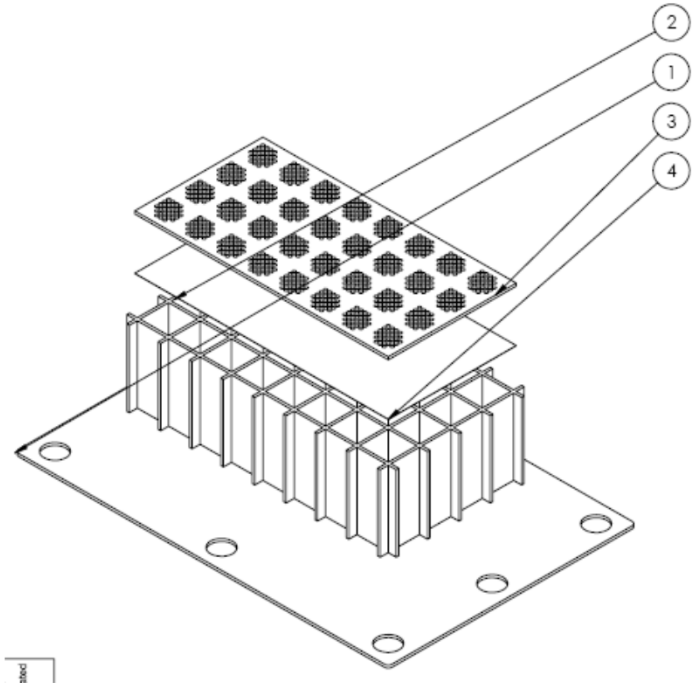

Impedance Eduction Results
9 mm cavity, 23 holes, plate thickness 0.6 mm, hole diameter 0.75 mm, porosity 0.20, no foam.

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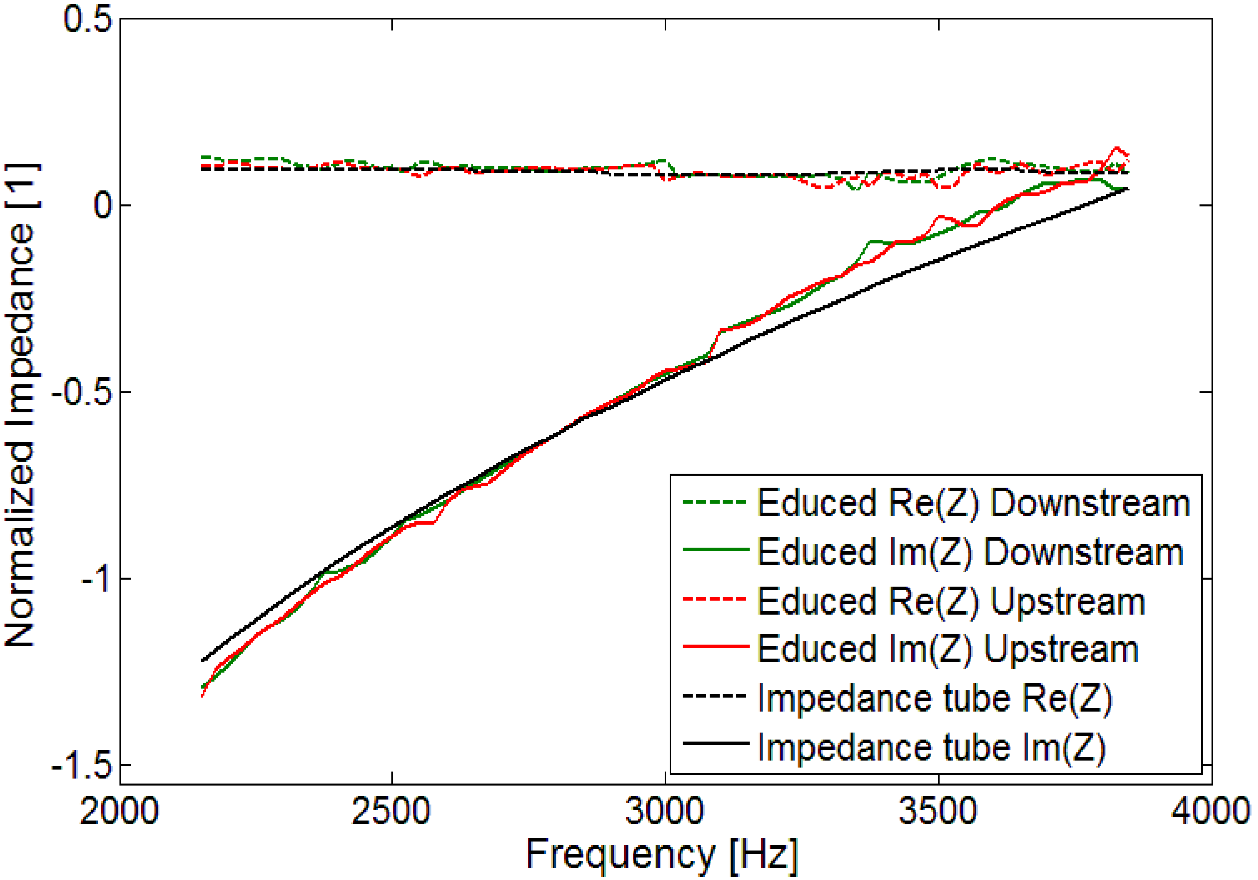

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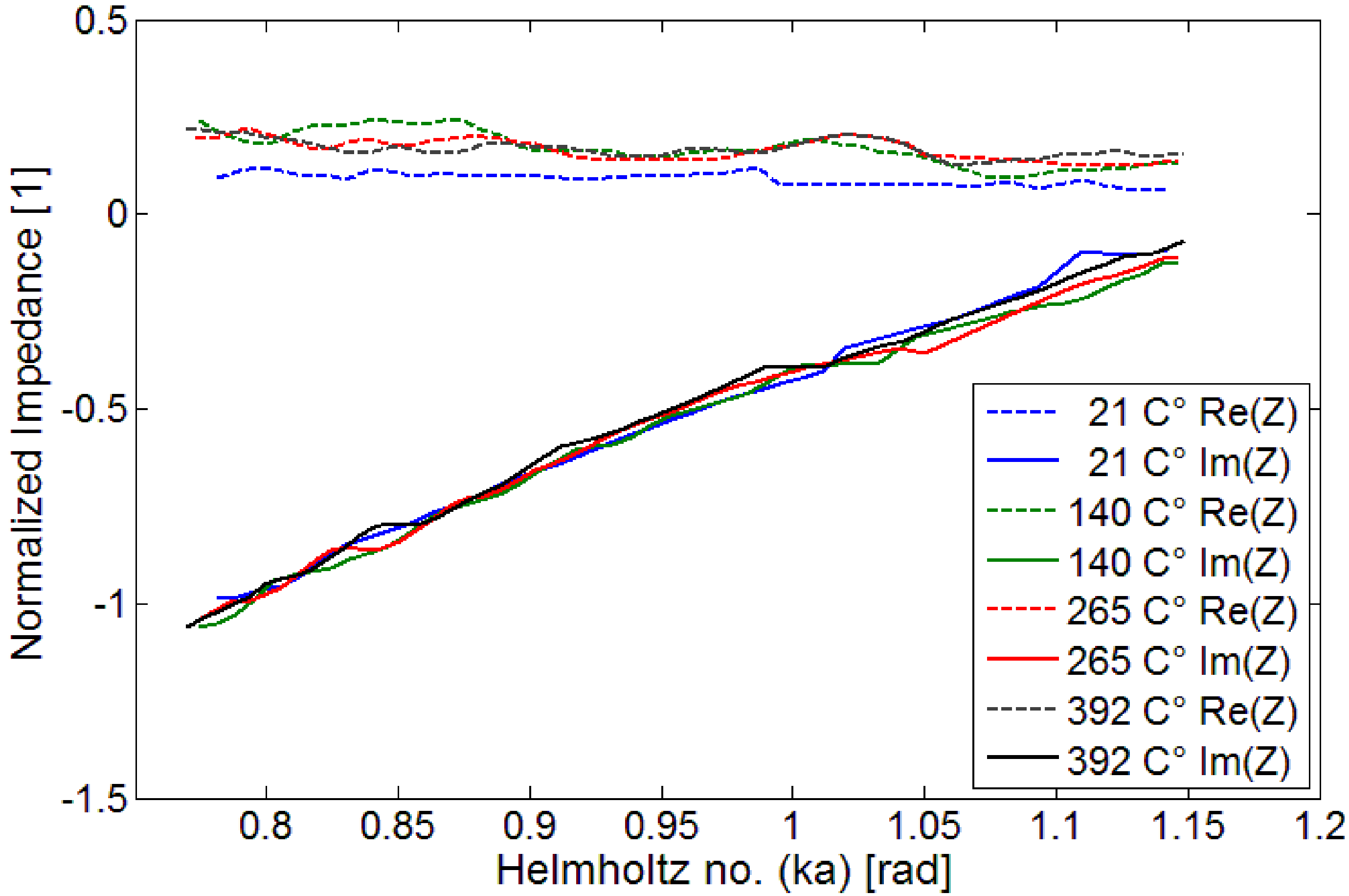

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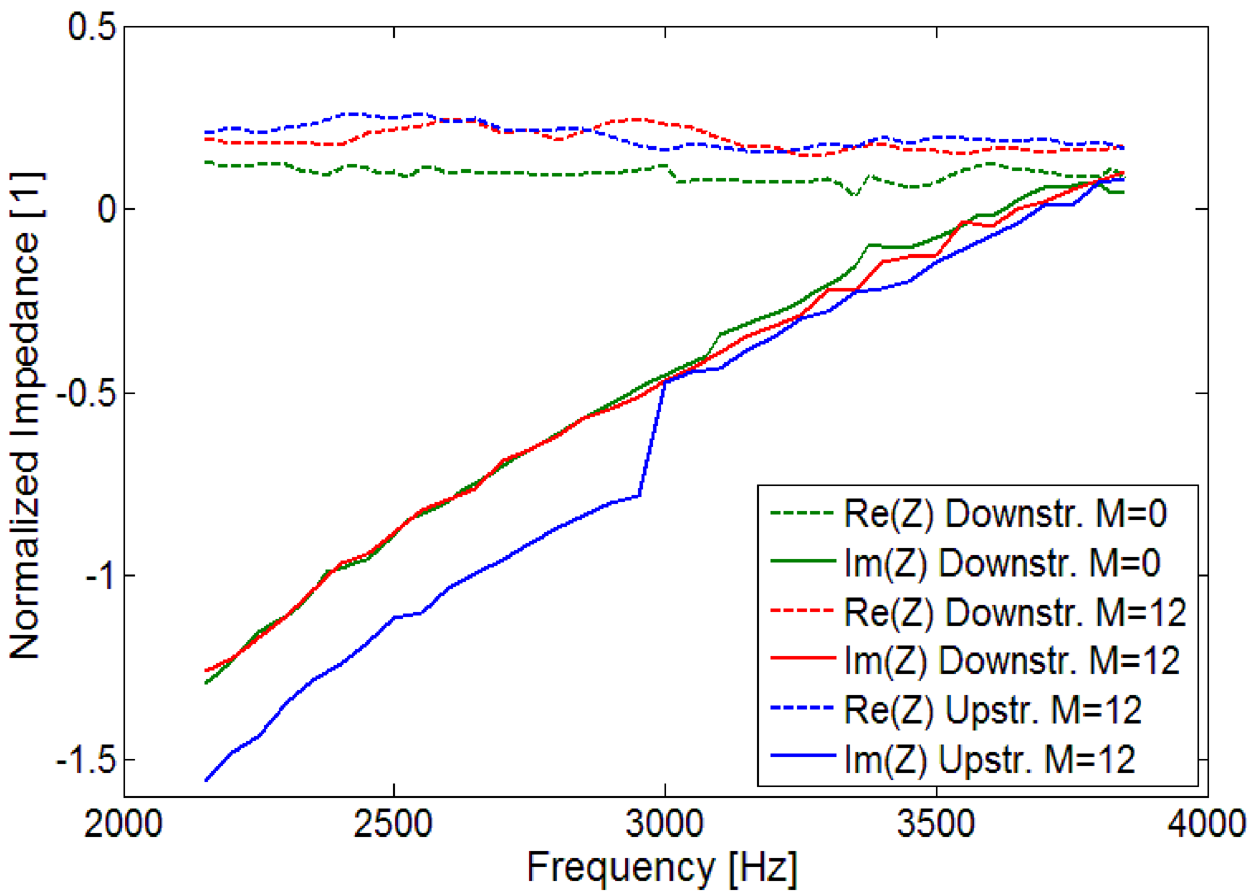

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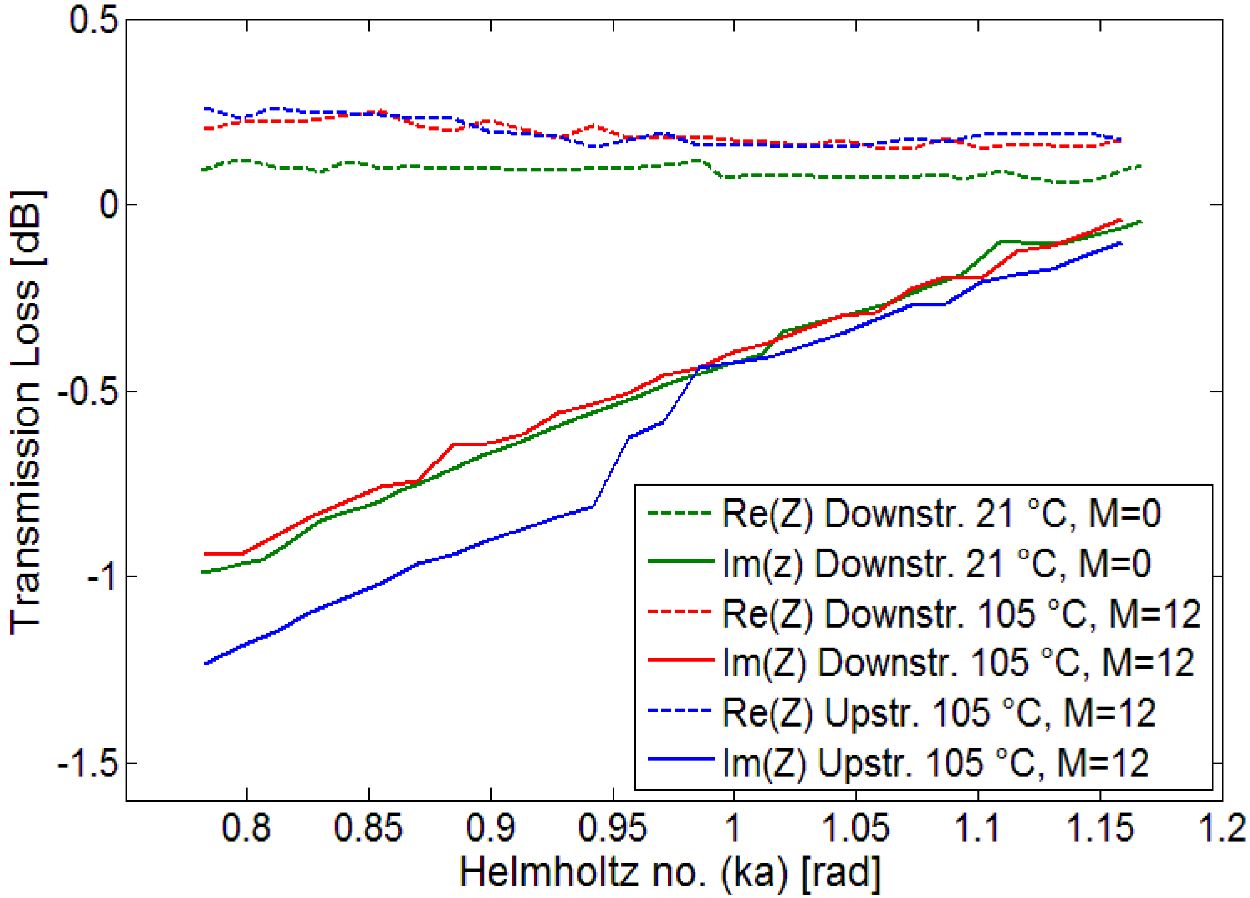

SUMMARY

The following topics were discussed:

- Passive one-port measurement techniques and the so-called two-microphone method. Sources of errors, the effect of test rig design and flow noise supression.
- Passive two-port measurement techniques including flow noise supression methods.
- Application to liner impedance eduction.

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